Do Real Exchange Rates Follow a Random Walk?
Extracted Inflation-Based Evidence from Japanese Yen

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Abstract

Using the pure price inflation rates extracted and estimated by an innovative financial-asset pricing method, we build and estimate univariate time series models of Japanese yen per U.S. dollar real exchange rates. Employing three methods of modeling, we find consistently that the extracted price index-based real exchange rate, r_t, obeys a *stationary*, mean-reverting process. The mean-reverting behavior detected is consistent with the less restrictive version of absolute PPP in which a real exchange rate is allowed to temporarily deviate from its mean. Further, the stationarity of r_t is fundamentally attributable to frequent and sharp changes in expectations reflected in goods prices that are implied by the extracted pure price inflation rates. Finally, an intuitive conjecture is presented that, in the long run, stationary series (r_t) would be *less* difficult to predict than nonstationary (random-walk) time series, because of the long-run, mean-reverting behavior of r_t. The conjecture is supported by the out-of-sample forecasting performance comparison between r_t and the random-walk CPI-based real exchange rate, r_t^{CPI}, for three- to six-month forecast horizons. This would reinforce evidence of the stationarity of r_t. A desirable feature of stationary real exchange rate is also presented with regard to equilibrium error.

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1 Introduction

The paper considers a set of three economic variables (all logged): a nominal exchange rate, s_t, of a home currency (Japanese yen) against a foreign currency (U.S. dollar), a foreign price index, p_t^*, and a home price index, p_t, where the price indices are constructed from the estimates for realized pure price inflation rate extracted from stock returns by Chowdhry, Roll and Xia (2005) (C-R-X). With extracted price index-based real exchange rate defined as $r_t \equiv s_t + p_t^* - p_t$, absolute purchasing power parity (PPP) asserts that $s_t + p_t^* = p_t$, implying $r_t = 0$, while relative PPP requires, in terms of percentage, that $\Delta s_t + \Delta p_t^* = \Delta p_t$ where Δ is the first difference operator.[1] C-R-X investigate the PPP puzzle, focusing on the failure of *relative* PPP, and successfully resolve the puzzle in the *short run* by using their extracted inflation rates as Δp_t^* and Δp_t (C-R-X, pp.260-261).

More recently, using the same extracted inflation rates, Kojima (2006), a companion paper, attempts in a vector error-correction (VEC) framework to explore the PPP relation and the impulse responses of prices and exchange rate, finding strong evidence supportive of the PPP restriction which yields *the equilibrium error in the form of an extracted price index-based real exchange rate,* $s_t + p_t^* - p_t$.

Two critical questions now arise: what model or stochastic process does the *extracted* price index-based real exchange rate (i.e., the equilibrium error in the PPP-based VEC model), r_t, obey, and is it stationary or not? The existing empirical literature on real exchange rate behavior is vast but, prior to C-R-X, only relies on the official price indices such as CPI and WPI.[2] The questions raised here are entirely new with regard to the real exchange rate under study, and twofold research objectives are thus set in the present paper:

(i) to model the real exchange rate, r_t; and

(ii) to derive and study implications of the model built.

For research objective (i), three methods of modeling are employed and combined so as to be assured of robustness of the results; an important past work relevant to the research objective is Roll (1979) presenting an

[1] See, for example, MacDonald and Marsh (1994, pp.24-25), and Hausman, Panizza and Rigobon (2006, p.94).

[2] The existing literature on real exchange rate (using official price indices) includes Glen (1992), Lothian and Taylor (1996), Wu (1996), and Cashin and McDermott (2004); for further past literature, see Kojima (1993) carrying out a time series analysis of the yen per dollar real exchange, focusing on time-series structure changes.

innovative, efficient markets view of the PPP that the real exchange rate should follow a random walk process. Research objective (ii) is designed to present implications that could help shed light on the stochastic features of the real exchange rate; in so doing, we focus on a less restrictive version of absolute PPP, equilibrium error (in the VEC context), expectations about future, and out-of-sample forecasting performance. In particular, a test of out-of-sample forecasting performance will be conducted in an attempt to reinforce the stochastic features of the model built in (i).

To my knowledge, the present paper is the very first attempt to apply C-R-X's inflation rates extracted from stock returns in investigating these topics for the yen per dollar rate, and we will proceed by analyzing a series of the following problems:

It is worthwhile first asking what graphical/visual features are observed in the inflation rate differentials between Japan and the U.S., and in the price index differentials. Exploring the features for both extracted and CPI inflations should help explain differences in movement, if any, between r_t and CPI-based real exchange rate, r_t^{CPI}, where $r_t^{CPI} \equiv s_t + cpi_t^* - cpi_t$ with cpi_t and cpi_t^* being, respectively, the Japanese and U.S. official CPIs.

Second, does the real exchange rate, r_t, follow a random walk? If it does not, then what stochastic process does it obey and is it stationary or not? To inquire consistency across modeling results, we will employ three methods of modeling r_t: unit-root testing, the Engle-Granger (1987) two-step cointegration test and the Box-Jenkins (1976, 1994) univariate time series modeling.

Third, as we derive implications of stochastic nature of r_t, we first look at a less restrictive version of absolute PPP and the concept of equilibrium error, and study the stationarity of the equilibrium error. After relating goods arbitrage and expectations about future to the stochastic nature of real exchange rates, we then turn to the out-of-sample forecasting performance test, asking, in particular, which real exchange rate is better predicted, r_t or r_t^{CPI}. This question is raised to examine our intuitive conjecture that, in the long run, stationary time series would be less difficult to predict than nonstationary series, because of the long-run, mean-reverting behavior of stationary series. If the conjecture is supported by the data, then evidence documented on the stochastic nature of r_t could be reinforced.

The remaining of the paper is organized as follows: section 2 gives the data source, and explores the visual features of r_t and r_t^{CPI} movements based on inflation rate differentials and price index differentials between Japan and the U.S. Section 3 models r_t by three mothods of modeling, and in section 4 some implications are derived that the stochastic nature of r_t yields with respect to absolute PPP, equilibrium error, expectations about future, and out-of-sample forecasting performance. Some concluding remarks, together with a summary of findings, are given in the final section.

2 Data and Real Exchange Rate Movement

2.1 Data

The data period is May 1983 through December 1999. All the data the present paper uses are detailed by Kojima (2006, section 2). The CPI inflation rates ($\pi^i_{CPI,t}, i = J, U$) and the estimates of pure inflation rates ($\hat{R}^i_{ft}, i = J, U$) that C-R-X extracted from the stock returns are plotted in Figures 1-4.[3] The U.S. and Japanese price indices (p_t^* and p_t), as drawn in Figure 5, are logs of price indices constructed from (and hence *implied* by) C-R-X's extracted inflation rates, with price indices at month April 1983 being set equal to unity. The Japanese and U.S. official CPIs (cpi_t and cpi_t^*), as drawn in Figure 7, are also studied and similarly constructed from their respective inflation rates.

The data sources for s_t, p_t^* and p_t (and for cpi_t and cpi_t^*) are now described. The source of the data for the yen per dollar exchange rate, s_t, is the same as that in C-R-X (pp. 261-262): the Database Retrieval System (v2.11), available at http://pacific.commerce.ubc.ca/xr/. The monthly percentage changes are computed between the *ends* of two adjacent months as $(s_t - s_{t-1}) \times 100$ with s_t denoting logged end-of-month exchange rate.

The official CPI inflation rates, as plotted in Figures 1-4, are also considered and their data source is also exactly the same as that used by C-R-X (p.261). While the official CPI inflation rates are being saved in one of the data files constructed by C-R-X and downloadable from the American Economic Association (AEA) Website, C-R-X's estimated pure inflation rates extracted from the stock returns are not and must

[3]Throughout the paper, multiplying the extracted inflation rates and CPI inflation rates by 100 gives percent-per-month figures.

be computed and saved by one of the program files downloaded from the AEA Website.[4]

Table 1 lists variable symbols used in the charts in the present paper, associating them with those used by C-R-X.

Table 1 Notation

Notation in the Graphs		Notation Following C-R-X[a]
eRf1, ERF1		$\hat{R}^J_{ft}$
eRf2, ERF2		$\hat{R}^U_{ft}$
gp1_c, GP1_C		$\pi^J_{CPI,t}$
gp2_c, GP2_C		$\pi^U_{CPI,t}$
p1, P1	p_t	log of $P^J_{R,t}$
p2, P2	p^*_t	log of $P^U_{R,t}$
p1_c	cpi_t	log of $P^J_{CPI,t}$
p2_c	cpi^*_t	log of $P^U_{CPI,t}$
e12, E12,	s_t	log of month-end yen per dollar exchange rate

[a]Superscripts, J and U, denote, respectively, Japan and U.S.

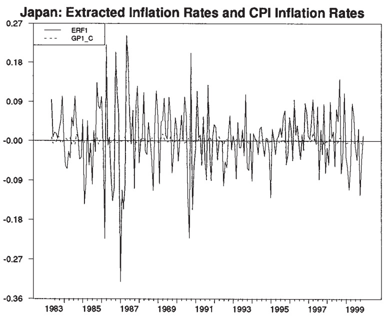

Figure 1 Japan: Extracted Inflation Rates $\hat{R}^J_{ft}$ (eRf1) and CPI Inflation Rates $\pi^J_{CPI,t}$ (gp1_c); 1983:5 - 1999:12. See also Table 1 for the notation.

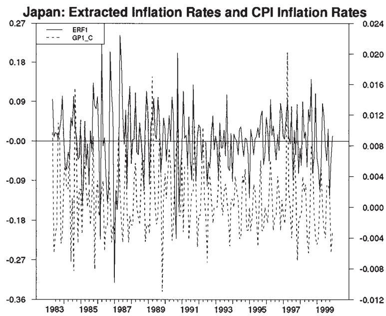

Figure 2 Japan: Extracted Inflation Rates and CPI Inflation Rates, with $\pi^J_{CPI,t}$ (gp1_c) on the Right-side Scale. See Figure 1 for the notation.

[4]See Kojima (2006, section 2) for details on C-R-X's extracted inflation rates.

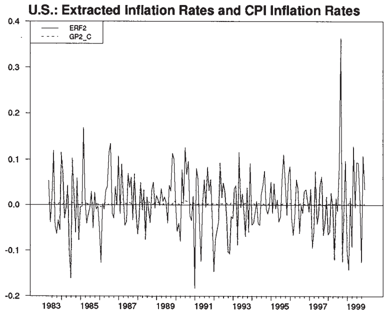

Figure 3 U.S.: Extracted Inflation Rates $\hat{R}^U_{ft}$ (eRf2) and CPI Inflation Rates $\pi^U_{CPI,t}$ (gp2_c); 1983:5 - 1999:12. See also Table 1 for the notation.

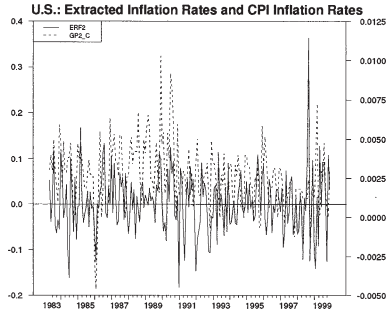

Figure 4 U.S.: Extracted Inflation Rates and CPI Inflation Rates, with $\pi^U_{CPI,t}$ (gp2_c) on the Right-side Scale. See Figure 3 for the notation.

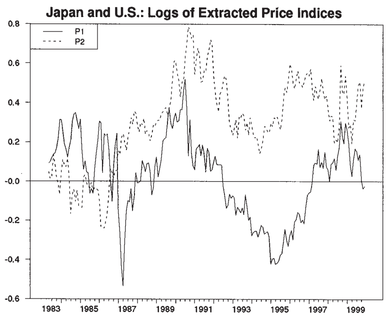

Figure 5 Japan and U.S.: Logs of Extracted Price Indices, p_t (p1) and p^*_t (p2); 1983:5 - 1999:12. See also Table 1 for the notation.

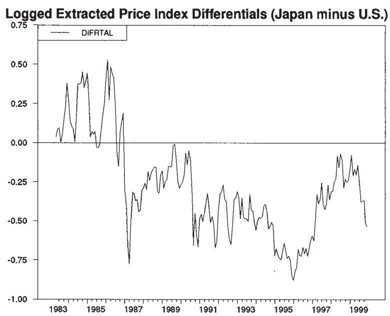

Figure 6 Logged Extracted Price Index Differentials (Japan minus U.S.): $p_t - p^*_t$ (p1 - p2); 1983:5 - 1999:12. See also Table 1 for the notation.

2.2 Movements of extracted price index- and CPI-based real exchange rates

Both inflation rate differentials and price index differentials between Japan and the U.S. are here studied visually to give possible explanations for differences in behavior between the two real exchange rates, r_t and r_t^{CPI}.

2.2.1 Inflation rate differentials between Japan and the U.S.

Ito (2005, pp.7-8) notes that "Since the [CPI] inflation rates of Japan and the U.S. have been very similar since the beginning of the 1980s, the tendency and fluctuations were parallel in the nominal and [CPI-based] real exchange rate. ... The real exchange rate movement broadly reflects the nominal exchange rate movement when inflation rates of the two countries are similar while the nominal exchange rate is volatile."[5]

Table 2 Summary Statitics:[a] Monthly Data From 1983:05 To 1999:12 (Observations=200)

Inflation Rate Differentials[b]	Mean	Std Dev	Minimum	Maximum
CPI: $\pi_{CPI,t}^{J} - \pi_{CPI,t}^{U}$ [c]	-0.0017	0.0050	-0.0131	0.0209
Extracted: $\hat{R}_{ft}^{J} - \hat{R}_{ft}^{U}$ [d]	-0.0019	0.1044	-0.4291	0.2883

[a]Not in percetage: multiplying $\hat{R}_{ft}^{i}$ and $\pi_{CPI,t}^{i}, i = J, U$ (inflation rate extracted from stock returns by C-R-X and CPI inflation rate, respectively) by 100 gives percent-per-month figures. The means and standard deviations match those (in percent per month) in C-R-X (Table 6, p.266).

[b]The same definition as given by C-R-X (Table 6, p. 266) is used.

[c]See Figure 10.

[d]See Figure 12.

"Very similar" CPI inflation rates of Japan and the U.S. as noted above are confirmed by comparing in Table 2 the standard deviations between the CPI inflation rate differentials, $\pi_{CPI,t}^{J} - \pi_{CPI,t}^{U}$, and the extracted inflation rate differentials, $\hat{R}_{ft}^{J} - \hat{R}_{ft}^{U}$ (or comparing the CPI inflation rate differentials in Figure 10 with the extracted inflation rate differentials in Figure 12). Notice how much *less* the CPI inflation rate differentials fluctuate.

This would imply that the extracted price index-based real exchange rate, r_t, should behave *very differently* from the nominal exchange rate, s_t, and hence from the CPI-based real exchange rate, r_t^{CPI}. This is

[5]The words in brackets [...] are inserted by the author, without changing in any way Ito's contention.

indeed confirmed by visual inspection of Figures 13 (for r_t) and 14 (for r_t^{CPI}); in the latter figure, the CPI-based real exchange rate is seen to be, as noted by Ito above, very similar in behavior to the nominal exchange rate.

2.2.2 Price index differentials between Japan and the U.S.

Interestingly enough, a roughly similar downward trend is observed in both CPI and extracted price index differentials, $cpi_t - cpi_t^*$ and $p_t - p_t^*$ (see Figures 6 and 8). This should be the case, since both indices are shown to be cointegrated by C-R-X. Note, however, that, while extracted price index and CPI are cointegrated, two real exchange rates computed based on them, r_t and r_t^{CPI}, exhibit sharply differing behavior due to distinct difference in volatility between extracted and CPI inflation rate differentials (as confirmed above by Table 2 and Figures 10 through 14).

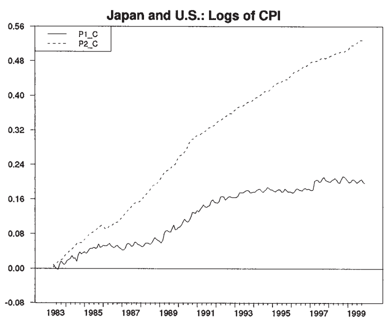

Figure 7 Japan and U.S.: Logged CPIs cpi_t (p1_c) and cpi_t^* (p2_c); 1983:5 - 1999:12. See also Table 1 for the notation.

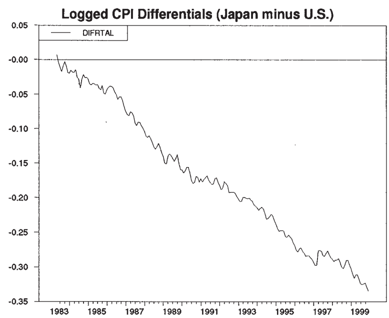

Figure 8 Logged CPI Differentials (Japan minus U.S.): $cpi_t - cpi_t^*$ (p1_c - p2_c); 1983:5 - 1999:12. See also Table 1 for the notation.

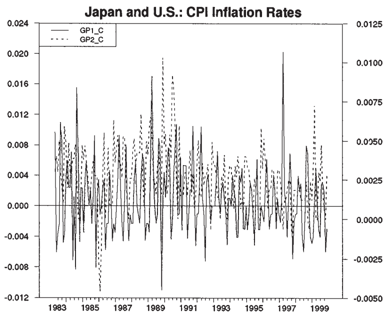

Figure 9 Japan and U.S.: CPI Inflation Rates $\pi^J_{CPI,t}$ (gp1_c) and $\pi^U_{CPI,t}$ (gp2_c), with $\pi^U_{CPI,t}$ (gp2_c) on the Right-side Scale; 1983:5 - 1999:12. See also Table 1 for the notation.

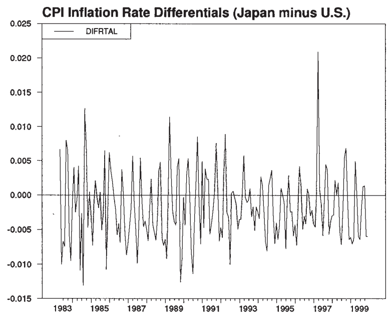

Figure 10 CPI Inflation Rate Differentials (Japan minus U.S.): $\pi^J_{CPI,t} - \pi^U_{CPI,t}$ (gp1_c - gp2_c); 1983:5 - 1999:12. See also Table 1 for the notation.

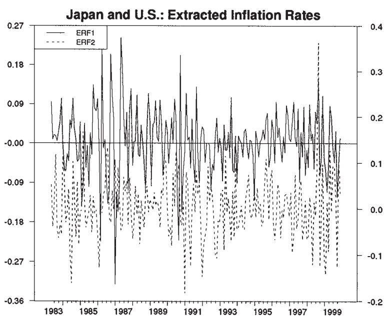

Figure 11 Japan and U.S.: Extracted Inflation Rates $\hat{R}^J_{ft}$ (eRf1) and $\hat{R}^U_{ft}$ (eRf2), with $\hat{R}^U_{ft}$ (eRf2) on the Right-side Scale; 1983:5 - 1999:12. See also Table 1 for the notation.

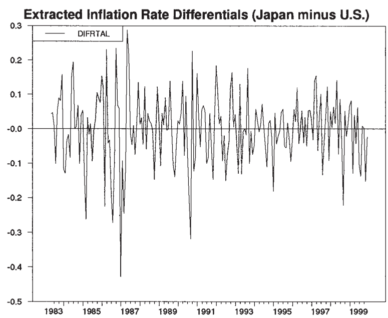

Figure 12 Extracted Inflation Rate Differentials (Japan minus U.S.): $\hat{R}^J_{ft} - \hat{R}^U_{ft}$ (eRf1 - eRf2); 1983:5 - 1999:12. See also Table 1 for the notation.

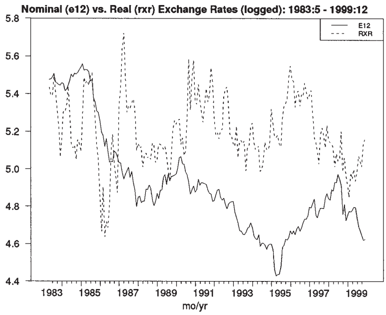

Figure 13 Nominal (s_t or e12) versus Real (r_t or rxr) Yen per Dollar Exchange Rates (logged): 1983:5 - 1999:12. See also Table 1 for the notation.

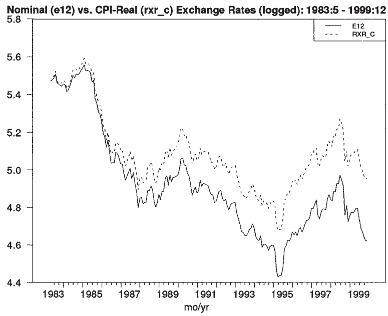

Figure 14 Nominal (s_t or e12) versus CPI-Based Real (r_t^{CPI} or rxr_c) Yen per Dollar Exchange Rates (logged): 1983:5 - 1999:12. See also Table 1 for the notation.

3 Time Series Modeling of Real Exchange Rate

To model the extracted price index-based real exchange rate, r_t (to test whether it obeys a random walk process, in particular), the present paper employs three approaches:[6]

(i) unit-root testing of r_t itself. The unit-root null (i.e., the null of a unit-root autoregressive process or a random walk) will be tested;

(ii) exploiting the Engle-Granger (single equation) two-step cointegration methodology. If the two series $q_t = s_t + p_t^*$ and p_t are *not* cointegrated, the three series, s_t, p_t^* and p_t, will wander away from one another in the long run, without settling down on an equilibrium track. This would then be an indication of the presence of random walk nature in the real exchange rate behavior; and

(iii) Box-Jenkins time series modeling method. Their model identification technique, in particular, provides a useful complementary way

[6] An additional approach is available to modeling real exchange rate (see MacDonald and Marsh, pp.25-28): (iv) estimating $\Delta r_t = c + \sum_{i=1}^{p} \phi_i \Delta r_{t-i} + u_t$. The random walk behavior will be assessed by testing the hypothesis that $\phi_i = 0$ for all i.

of examining stationarity/nonstationarity, based on sample autocorrelations and partial autocorrelations. That is, if sample autocorrelations suggest nonstationarity of r_t, then first-order differencing will be needed (before estimating a time series model) so that Δr_t will become stationary. Otherwise (i.e., if r_t is found stationary to begin with), no differencing will be required before estimation.

The test results from all three methods, (i) through (iii), will be combined in an attempt to obtain a consistent modeling result. In particular, *non*cointegration in (ii) should imply a random-walk real exchange rate, if methods (i) and (iii) both lead to a nonstationary time series model for r_t.

3.1 Unit-root testing

The top panel of Table 3 reports *strong* rejection (at the 1% level) of the unit-root null for the extracted price-based real exchange rate itself,[7] suggesting that r_t is *not* nonstationary and does not seem to follow a random walk process. This coincides with our visual observation of Figure 13 that there appears, for example, no definite trend in r_t.

One remark is in order on the *non*stationarity of the CPI-based real exchange rate, r_t^{CPI}. The middle panel of Table 3 reports the unit-root test result for r_t^{CPI} plotted in Figure 14: the unit-root null is *not* rejected, as consistent with the figure (in which r_t^{CPI} is seen to exhibit a trend similar to the nominal exchange rate) and as widely documented in the existing literature.[8] (A time series model of r_t^{CPI} will be estimated later and reported in Table 12, which will be seen to accord with non-rejection of the unit-root null here.)

[7]Notice that the null is decisively rejected here, *despite* the low power against stationary alternatives that standard univariate unit-root tests are known to have in finite samples. See Rudebusch (1993, pp.264-265) for the early research questioning the ability of the unit-root tests to reject the unit-root null when it is indeed false (i.e., stationary alternatives are true); if the unit-root null *failed* to be rejected, then it could be mainly due to the low power of the tests. (A more powerful, panel data-based testing procedure is proposed by Wu 1996, pp.55-58, to test for the long-run PPP. The proposed test will not be conducted here, for we only study Japanese yen exchange rate.)

[8]See the footnote immediately above on failure to reject the unit-root null.

Table 3 Unit-Root Tests[a] for Residuals from Static Models (1), and for Real Exchange Rate, r_t, and CPI-Based Real Exchange Rate, r_t^{CPI}

	Dickey-Fuller[b]	Phillips-Perron[c]	
With intercep only,[d] with	12 lags		
With intercept and trend with		4 lags in the error process	12 lags in the error process
T-test statistic: Real exchange rate, r_t	−3.849[e]	−4.460	−4.295
With intercept and trend,[f] with	12 lags		
With intercept and trend with		4 lags in the error process	12 lags in the error process
T-test statistic: CPI-based real exchange rate, r_t^{CPI}	−2.464[g]	−1.882	−1.901
Regression Run	1984:06 to 1999:12	1983:10 to 1999:12	1984:06 to 1999:12
Observations	188	200	200
Without intercep and trend[h] with	12 lags		
With intercept and trend with		4 lags in the error process	12 lags in the error process
T-test statistic: Residuals	−3.518[i]	−4.013	−3.926

[a]See Dickey and Fuller (1979), Phillips (1987) and Phillips, and Perron (1988). For the difference between the tests see Doan (*UG*, p.242); for inclusion of the trend term, see Doan (*UG*, p.246).

[b]The general form of univariate ADF regression is $\Delta y_t = \sum_{l=1}^{p} \phi_l^{\Delta} \Delta y_{t-l} + \alpha y_{t-1} + \mu + \psi t + u_t$ where Δ is the first difference operator. The null of a unit root is $\alpha = 0$. The ADF statistics adjust for autocorrelation using an autoregressive representation.

[c]The PP statistics correct for autocorrelation using a non-parametric correction. Critical values that apply here are: 1%= −3.464, 5%=−2.876, 10%= −2.574.

[d]Here it is more appropriate for the constant to be included, for the real exchange rate has a nonzero mean, without any obvious trend (see Figure 13).

[e]Critical values are 1%= -3.466 5%= -2.877 10%= -2.575.

[f]Here it is more appropriate for the constant and trend to be included, for the CPI-based real exchange rate has a nonzero mean with a trend (see Figure 14).

[g]Critical values are 1%= -4.010 5%= -3.435 10%= -3.141.

[h]Both intercept and trend are omitted. For a residual sequence, there is no need to include an intercept term in the ADF test (Enders 2004, pp.336). There is no such an option for the PP test in Doan (*RATS*).

[i]Critical values that apply here are: 1%= −2.577, 5%= −1.941, 10%= −1.617. (Including the constant for the residuals yields T-test statistic -3.509 with Critical values: 1%= -3.466 5%= -2.877 10%= -2.575, which again leads to the same decision.)

3.2 Engle-Granger (single equation) two-step cointegration test

3.2.1 Two-step test

Step 1: Unit-root tests for the price series It is shown elsewhere, as remarked in Kojima (2006, section 3.2.1), that the augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test cannot reject, at any conventional levels of significance, the null of a unit root for each of p_t, cpi_t, p_t^* and cpi_t^*. The same non-rejection of the null is also documented for the nominal exchange rate, s_t.[9]

Step 2: Unit-root tests for the residuals, to check up on a cointegration: a static approach[10] The Engle-Granger *static*, long-run model is estimated by an OLS method, with β_0 and β_1 being the cointegrating or long-run parameters:

$$y_t = \beta_0 + \beta_1 x_t + v_t, \tag{1}$$

with $y_t = s_t + p_t^*$, and $x_t = p_t$. The regression results are reported in Table 4, and the regression line and residuals are plotted, respectively, in Figures 15 and 16. The unit-root test for the residuals is carried out in the bottom panel of Table 3, to check up on a cointegration: both the ADF and PP tests reject the null of a unit root, *decisively* at the 1-percent level.[11] The residual sequence thus is stationary, which, together with s_t, p_t^* and p_t being all integrated of order one, implies that $s_t + p_t^*$ and p_t are cointegrated. (Though not reported, exactly the same results are obtained for s_t versus the price index differentials $p_t - p_t^*$, based on the figures and table very similar to Figures 15, 16 and Table 3.)

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[9]Those who challenge the unit-root tests would argue that failure to reject the unit-root nulls is mainly due to the low power of the tests (see the earlier footnote). Still, we will not question the unit-root test results here, since they are consistent with both Kojima (2006, Table 2) who rejects by the (multivariate) Johansen procedure the null of *stationarity* for each of s_t, p_t and p_t^*, and C-R-X (p.274) who show that all p_t, cpi_t, p_t^* and cpi_t^* appear to be integrated of order one, $I(1)$.

[10]The dynamic approach as recommended by Harris (1995, p.72) is not considered in the present paper.

[11]Again, the null is strongly rejected, despite the low power of univariate unit-root tests.

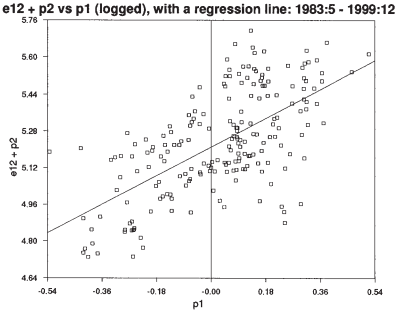

Figure 15 $s_t + p_t^*$ (e12+p2) versus p_t (p1): 1983 : 5 – 1999 : 12. See also Table 1 for the notation.

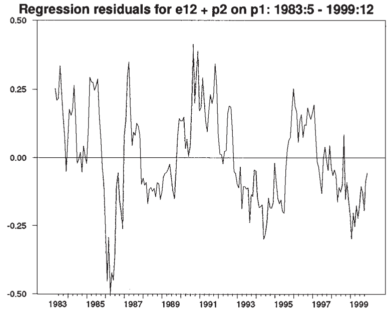

Figure 16 Residuals from Model (1): 1983:5 - 1999:12. The regression line is drawn in Figure 15. See also Table 1 for the notation.

3.2.2 Implications

In the long run the yen per dollar exchange rate and the extracted price indices are comoving with one another, which means that these time series would *not* wander arbitrarily far from each other. This in turn implies that the extracted price index-based real exchange rate defined as $s_t + p_t^* - p_t$ would *not* be nonstationary and hence not follow a random walk process.[12] This implication does coincide with the decisive rejection of the unit-root null for the real exchange rate, r_t, itself (as reported in the top panel of Table 3).

If the real exchange rate was not a random walk, what process or model would it obey? The next section attempts to model r_t by the Box-Jenkins univariate time series method, and the stationarity of r_t will be again examined.

3.3 Box-Jenkins modeling

Following the Box-Jenkins univariate modeling method, it is shown in Figure 17 (especially, the left pair of SACF (sample autocorrelation func-

[12]Note that the *lack* of cointegration suggests that the time series would wander arbitrarily far from each other, in which case nonstationary behavior would result.

tion) and SPACF (sample partial autocorrelation function)) that the real exchange rate series appears stationary so that there will be no need for differencing of any order before estimation,[13] and that the appropriate time series model will be identified as a first-order autoregressive model, AR[1]:[14]

$$r_t = c + \phi_1 r_{t-1} + u_t. \tag{2}$$

The estimated AR[1] model (2) is reported in Table 5 and its graphical diagnostic check, relying on Hokstad (1983), is provided in Figure 19. The model seems adequate: in particular, no additional AR or moving average (MA) parameter is needed (as noted in Figure 19).

Is the stationarity condition that the first-order AR parameter ϕ_1 be less than unity in absolute value satisfied? On grounds of the two earlier test results rejecting *firmly* nonstationarity of r_t (that is, the decisive rejection of a random-walk null and the Engle-Granger cointegration test result that $s_t + p_t^*$ and p_t are comoving), we would infer that ϕ_1 of 0.834 (in Table 5) is less than unity, and thus that the stationarity condition is satisfied.

Yet, those who hold strong a priori information that real exchange rates are random walk may *not* be persuaded otherwise by the results above. We thus, next, present implications that stationary (extracted price-based) real exchange rate has with respect to absolute PPP, equilibrium error, expectations about future, and out-of-sample forecasting performance. Those implications should help shed light on the stationary features of the real exchange rate, r_t, in contrast to the nonstationary behavior of r_t^{CPI}.

---

[13]Even without being differenced, r_t looks stationary, since in Figure 17 the (left) SACF of r_t decays quite rapidly as lag becomes larger. How rapidly it is is clear when compared with the (left) SACF in Figure 18 for the CPI-based real exchange rate which is shown earlier in Table 3 to be nonstationary (having a unit root). That is, the SACF of r_t remains statistically significant only up to lag 5, while that of r_t^{CPI} continues to be significant till as far as around lag 16. Following the Box-Jenkins time-series modeling method, then, r_t will be stationary (and hence need not be differenced), whereas r_t^{CPI} is nonstationary (and hence need be differenced to be made stationary). See, for example, Kojima (2005, Figure 5) who draws SACFs and SPACFs of various stationary time series generated by simulation.

[14]It is identified as a first order AR, again by the Box-Jenkins method, since in Figure 17 the (left) SPACF has a statistically significant spike only at lag one. See, again, Kojima (2005, Figure 5) for details on how to identify a univariate time series model by inspection of its SACF and SPACF.

Table 4 Linear Regression - Estimation by Least Squares: Model (1)

Dependent Variable	$s_t + p_t^*$
Constant	5.211 (0.000)[a]
p_t	0.695 (0.000)
Monthly Data	1983:05 To 1999:12
Usable Observations	200
Degrees of Freedom	198
Adjusted R^2	0.408
Residual Standard Deviation	0.174
Regression F(1,198)	138.081 (0.000)
Durbin-Watson Statistic	0.264

[a]P-value.

Table 5 Box-Jenkins - Estimation by Gauss-Newton: AR[1] Model (2)

Dependent Variable	r_t
Constant	5.195 (0.000)[a]
r_{t-1}	0.834 (0.000)
Monthly Data	1983:06 To 1999:12
Usable Observations	199
Degrees of Freedom	197
Adjusted R^2	0.699
Residual Standard Deviation	0.101
Regression F(1,197)	460.779 (0.000)
Residuals:[b]	
Durbin-Watson Statistic	1.878
Q(36-1)	21.583 (0.963)

[a]P-value.

[b]For a further graphical residual analysis, see Figure 19.

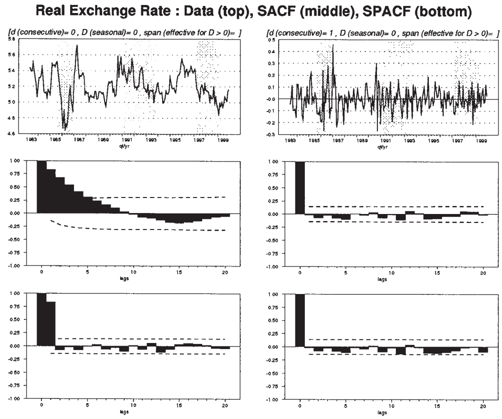

Figure 17 Real Yen per Dollar Exchange Rate, r_t. SACF = sample autocorrelation function; SPACF = sample partial autocorrelation function; the left set of graphs is for r_t itself, and the right set for its first-differenced series.

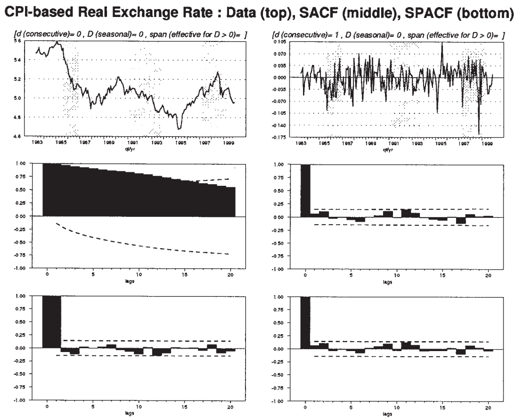

Figure 18 CPI-Based Real Yen per Dollar Exchange Rate, r_t^{CPI}. See Figure 17 for notes.

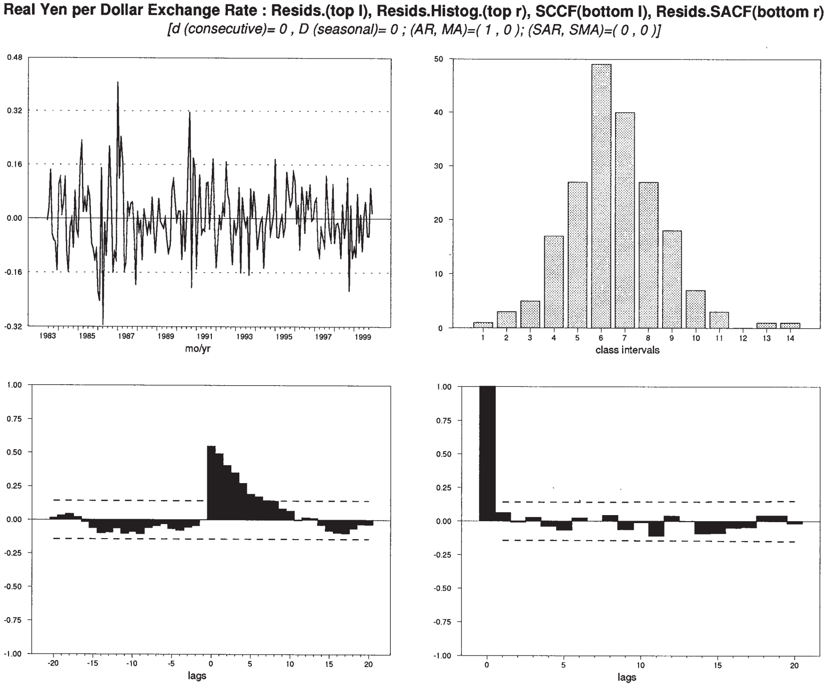

Figure 19 Real Yen per Dollar Exchange Rate, r_t: AR[1] Residual Analysis. The SCCF (sample cross correlation function) at lag l is SCCF between data(t) and resids($t-l$): statistically significant SCCF at a lag $l < 0$ suggests a need for an additional AR term at l. Statistically significant residual SACF (sample autocorrelation function) at a lag l suggests a need for an additional MA term at l. There is neither need detected here. (The diagnostic check here relies on Hokstad 1983).

4 Implications of Stationary Real Exchange Rate

4.1 PPP and equilibrium error

4.1.1 Less restrictive version of absolute PPP

With ϕ_1 less than unity in AR[1] model (2), *short-run* deviations from PPP are present but will be only temporary and corrected at a rate equal to $1-\phi_1$, eventually disappearing in the *long run*: the real yen exchange rate, r_t, is *mean reverting* where the mean is c (=5.195 in Table 5).[15] This is indeed consistent with the less restrictive version of absolute PPP in which a real exchange rate is allowed to temporarily deviate from its mean, and, interestingly enough, short-run deviations from PPP here correspond to the concept of equilibrium error, as will be seen next.

4.1.2 The equilibrium error

Engle and Granger (1987) defines equilibrium error as follows: a set of n economic variables ($y_{it}, i = 1, ..., n$) is in long-run equilibrium when

$$\sum_{i=1}^{n} \beta_i y_{it} = 0; \tag{3}$$

the equilibrium error is a disequilibrium defined as a deviation from long-run equilibrium and given by

$$e_t \equiv \sum_{i=1}^{n} \beta_i y_{it}; \tag{4}$$

in the long run,

$$e_t = 0. \tag{5}$$

Clearly, real exchange rate, $r_t \equiv s_t + p_t^* - p_t$, takes the form of eq. (4) with $\beta_i = 1, i = 1, 2$ and $\beta_3 = -1$, and thus may be interpreted as an

[15] See MacDonald and Marsh (p.25). Note that, using price *levels* instead of price indices, *long-run* PPP requires that $c = \phi_1 = u_t = 0$ in eq. (2), that is, r_t must be a zero-mean stationary process; when using price *indices*, as in the present study, r_t may be equal to the (non-zero) constant, c. See again Figure 13 for the mean-reverting movement of r_t.

equilibrium error; the equilibrium error here is stationary, since r_t has been shown to be stationary.

The stationarity of equilibrium error is an important result for a vector error-correction (VEC) form (7) of the vector autoregressive (VAR) model (6), as is now shown. With $\boldsymbol{y}_t = (s_t, p_t^*, p_t)'$, each element of which is a potentially endogenous variable and assumed to be integrated of order 1, $I(1)$, Kojima (2006) considers VAR model including a constant and augmented with centered seasonal dummies:

$$\boldsymbol{y}_t = \sum_{l=1}^{L} \boldsymbol{\Phi}_l \boldsymbol{y}_{t-l} + \boldsymbol{\mu} + \boldsymbol{\Psi} \boldsymbol{D}_t + \boldsymbol{u}_t. \tag{6}$$

The underlying VAR model is reformulated in an error-correction form as the VEC model:

$$\Delta \boldsymbol{y}_t = \sum_{l=1}^{L-1} \boldsymbol{\Phi}_l^{\Delta} \Delta \boldsymbol{y}_{t-l} + \boldsymbol{\Pi} \boldsymbol{y}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\Psi} \boldsymbol{D}_t + \boldsymbol{u}_t \tag{7}$$

where the *short-run* matrices $\boldsymbol{\Phi}_l^{\Delta}$ represent the short-run dynamics/adjustment to past change, $\Delta \boldsymbol{y}_{t-l}$, and the *long-run* marix $\boldsymbol{\Pi}$ represents long-run adjustment.[16] If the long-run marix $\boldsymbol{\Pi}$ is non-zero but less than full-rank (letting r denote the rank), then it is usefully written as $\boldsymbol{\Pi} = \boldsymbol{\alpha\beta}'$ and hence, in (7), $\boldsymbol{\Pi y}_{t-1} = \boldsymbol{\alpha\beta}' \boldsymbol{y}_{t-1}$ with $\boldsymbol{\beta}' \boldsymbol{y}_{t-1}$ being equilibrium error, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $3 \times r$ matrices.[17]

Kojima (2006) estimates the VEC model (7) of prices and the yen per dollar exchange rate, finding strong evidence supportive of the PPP restriction which yields the equilibrium error, $\boldsymbol{\beta}' \boldsymbol{y}_{t-1} = r_{t-1} (\equiv s_{t-1} + p_{t-1}^* - p_{t-1})$. As a consequence, the stationarity of the real exchange rate results in the equilibrium error being stationary. This is a desired result for the VEC model (7), since one of the requirements for the model is that $\boldsymbol{\Pi y}_{t-1}$, in which the equilibrium error ($\boldsymbol{\beta}' \boldsymbol{y}_{t-1}$ or, equivalently, $s_{t-1} + p_{t-1}^* - p_{t-1}$) is embedded, must be stationary for $\boldsymbol{u}_t \sim I(0)$ to be white noise.

[16] The initial assumptions include, in particular, the white noise, $\boldsymbol{u}_t \sim IN(\boldsymbol{0}, \boldsymbol{\Sigma})$ or $\boldsymbol{u}_1, ..., \boldsymbol{u}_T$ are $niid(\boldsymbol{0}, \boldsymbol{\Sigma})$; the dependence is allowed among the white-noise disturbance terms, $u_{1t_1}, u_{2t_2}, u_{3t_3}$ for any $t_i, i = 1, 2, 3$.

[17] $\boldsymbol{\alpha}$ is a matrix representing a measure of the average speed of convergence towards the long-run equilibrium (i.e., the speed of adjustment to disequilibrium).

4.2 Expectations and goods arbitrage

The extracted price index-based real yen exchange rate follows a stochastic process (2), different from a random walk process advocated by Roll (1979) in his efficient markets view of PPP, which is derived using parity conditions from the capital account of the balance of payments. The difference in stationarity here may be fundamentally attributable to expectations reflected in goods prices.

The rationale behind real exchange rate being *non*stationary includes (i) sluggish changes in such real factors as productivity, technology and consumer preferences. As real factors vary slowly over time (in the long run), they will change only in a sluggish manner the relative prices of goods between home and foreign countries, thus likely making the price differentials between the two countries stable (i.e., not too volatile) and moving the real exchange rate towards some direction, for a certain period of time.

This tendency in a certain direction for a certain period of time may also be, in part, due to (ii) goods being not freely traded.[18] That is, reflected in the goods prices are "more *present* and *past* circumstances as they are embedded in existing contracts" (than expectations about *future* circumstances),[19] and thus the arbitrage speed is very slow in the goods markets. This in turn makes sticky and less fluctuate the price differentials between two countries, and, as a result, (ii) as well as (i) most likely induce nonstationarity (or a tendency in a certain direction for a certain period of time) of real exchange rate. Such widely documented, nonstationary real exchange rate is the one computed based on slow moving, official price indices.[20]

On the other hand, if goods are freely traded between two countries and arbitrage in goods markets is as fast and continuous as that in financial markets,[21] frequent and sharp changes in expectations about future circumstances would be reflected in goods prices (as well as in asset prices like exchange rates). This further would make the price

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[18] See, for example, C-R-X (pp.255-256).

[19] See, for example, Frenkel (1981, p.162).

[20] See r_t^{CPI} in Figure 14 for its behavior (as compared with the r_t behavior in Figure 13). Later, section 4.3.2 again looks at a tendency in a certain direction for a certain period of time as exhibited by the official price-based real exchange rate.

[21] See, for example, Hausman, Panizza and Rigobon (p.94).

differentials between two countries sufficiently volatile[22] that the real exchange rate would not tend toward any direction but rather move in an erratic manner.

An example of real exchange rate exhibiting such an erratic behavioral pattern may be the real exchange rate, $s_t + p_t^* - p_t$, with the price indices being those implied by C-R-X's inflation rates, $\hat{R}_{ft}^J$ and $\hat{R}_{ft}^U$, extracted from stock returns. As we already saw in Table 2 and the figures referred to there, C-R-X's extracted inflation rate differentials are highly volatile between Japan and the U.S. This implies that those pure price inflation rates estimated by C-R-X successfully capture fast and frequent goods arbitrage that would not be otherwise observed by the slow-moving, official CPI inflation rate. As a result, the extracted price index-based real yen exchange rate would be expected to behave in an erratic manner (possibly around a certain level which in the long run could be its mean),[23] and such a behavior is indeed found in section 3 consistent with stationarity.[24]

4.3 Out-of-sample forecasting performance

A test of out-of-sample forecasting performance could also help shed light on the stationary features of the extracted price index-based real exchange rate, r_t, as compared with the random-walk behavior of CPI-based real exchange rate, r_t^{CPI}. We ask a question: which real exchange rate is better predicted, r_t or r_t^{CPI}? The answer will naturally depend on the length of out-of-sample forecast horizon, and our intuitive conjecture is that, in the long run, stationary series (r_t) would be *less* difficult to predict than nonstationary (random-walk) time series (r_t^{CPI}), because of the long-run, mean-reverting behavior of r_t. If this conjecture is supported by the data, then evidence as presented earlier (in section 3) of a stationary, mean-reverting nature of r_t and a random-walk, wandering feature of r_t^{CPI} would be, in effect, reinforced. No conjecture is pre-

[22]For evidence on the high volatility, see Figure 6 (to be contrasted with Figure 8).

[23]See r_t in Figure 13 for its erratic behavior (as compared with the r_t^{CPI} behavior in Figure 14).

[24]Consistency with stationarity here could also be studied by asking whether the extracted price index-based real exchange rate satisfies standard stationarity conditions that its mean, variance and autocovariances are all time-invariant, with autocovariances dependent only on lag. Instead, we will confirm the consistency later in section 4.3.2, by visually looking at the erratic nature of the stationary extracted price-based real exchange rate.

sented with regard to forecasting performance in the short run, and we will investigate, by way of forecast statistics check, whether short-run forecasting performance is consistent with stationarity of r_t and nonstationarity of r_t^{CPI}.

We also ask which model better predicts r_t, AR[1] model (2) or a random walk model

$$r_t = c + r_{t-1} + u_t. \tag{8}$$

The forecasting performance test result for the question will be presented and interpreted later at the end of the present section.

Our final exercise is now to compute forecast statistics and test the long- and short-run, out-of-sample forecasting performance of the real exchange rate models. The sample period is now from May 1983 up to June 1999, and the out-of-sample period July 1999 through December 1999.

4.3.1 Forecasting performance statistics

Extracted price index-based real exchange rate We forecast the level of extracted price index-based real exchange rate, r_t, by AR[1] model (2). The estimated model is presented in Table 6 for the sample period up to June 1999; the estimated result is almost the same as that for the sample period up to December 1999 in Table 5. The dynamic out-of-sample forecasts are tabulated in Table 7, and plotted in Figure 20.[25]

The forecasting performance statistics are computed based on a different set of forecasts than that in Table 7: Doan (*RATS*) re-estimates the AR[1] model by extending the sample period and accordingly shortening the forecast period; the re-estimated results are reported in Table 8, with the corresponding, *updated* forecasts in Table 9.[26] The forecasting performance statistics computed based on those updated forecasts in Table 9 are shown in the top panel of Table 10. In the table, Theil's U statistics below [above] 1 indicates that the AR[1] model's RMSE is [fails to be] smaller than that for the naive, no-change (i.e., flat) forecasts which are being set equal to the value at the forecast origin. The Theil's U

[25] Dynamic forecasts are multi-step forecasts, where forecasts computed at earlier horizons are used for the lagged dependent variable terms at later horizons: for example, the forecasted value computed for time T will be used as the first-period lag value for computing the forecast at time $T+1$, and so on (Doan, *UG*, p.287).

[26] The re-estimation process here is detailed by Kojima (2005, pp.77-78).

statistics (of between 0.255 and 0.800) in the top panel suggest that the AR[1] model performs better than the naive model, for all the months ahead.

The forecasting performance statistics here will be later compared with the counterparts for the CPI-based real exchange rate model, over the long and short forecast horizons.

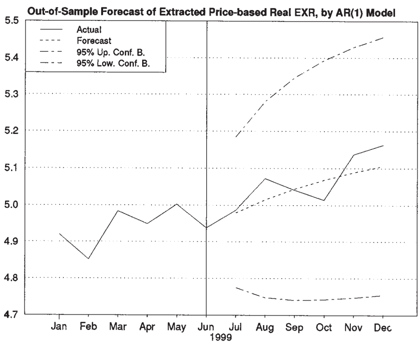

Figure 20 Out-of-sample Forecasts of Real Exchange Rate r_t by AR[1] Model (2), with Forecast Period of 1999:7 - 1999:12. The forecasts are reported in Table 7; standard errors for the 95% confidence bands plotted are computed following Doan (*RM*, pp.122-123).

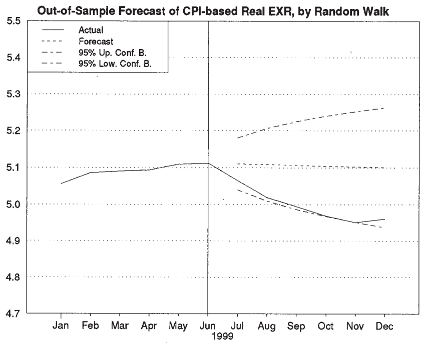

Figure 21 Out-of-sample Forecasts of CPI-Based Real Exchange Rate r_t^{CPI} by Random Walk Model (9), with Forecast Period of 1999:7 - 1999:12. The forecasts are reported in Table 13; also plotted are the 95% confidence bands (see note in Figure 20).

Table 6 Box-Jenkins - Estimation by Gauss-Newton: AR[1] Model (2)[a]

Dependent Variable	r_t
Constant	5.193 (0.000)[b]
r_{t-1}	0.836 (0.000)
Monthly Data	1983:06 To 1999:06
Usable Observations	193
Degrees of Freedom	191
Adjusted R^2	0.695
Residual Standard Deviation	0.102
Regression F(1,191)	439.489 (0.000)
Residuals:	
Durbin-Watson Statistic	1.874
Q(36-1)	21.814 (0.960)

[a]Convergence in 3 Iterations. Final criterion was 0.0000000 < 0.0000100.

[b]P-value.

Table 7 Out-of-sample Forecasts of r_t:[a] Forecast Period From 1999:07 To 1999:12

1999:07	4.980
1999:08	5.015
1999:09	5.044
1999:10	5.068
1999:11	5.089
1999:12	5.106

[a]The (dynamic) forecasts are computed by the estimated model in Table 6.

Table 8 Re-estimation Results for Updating the Forecasts in Table 9

Re-estimation:	First	Second	Third
Dependent Variable	r_t	r_t	r_t
CONSTANT	5.193 (0.000)[a]	5.195 (0.000)	5.193 (0.000)
r_{t-1}	0.836 (0.000)	0.834 (0.000)	0.835 (0.000)
Monthly Data	1983:06 To 1999:07	1983:06 To 1999:08	1983:06 To 1999:09
Usable Observations	194	195	196
Degrees of Freedom	192	193	194
Adjusted R^2	0.697	0.698	0.699
Residual Standard Deviation	0.102	0.102	0.102
Durbin-Watson Statistic	1.878	1.874	1.878
Q(36-1)	21.955 (0.958)	21.972 (0.958)	21.388 (0.966)
Re-estimation:	Fourth	Fifth	Sixth (Final)[b]
Dependent Variable	r_t	r_t	r_t
CONSTANT	5.191 (0.000)	5.195 (0.000)	5.195 (0.000)
r_{t-1}	0.836 (0.000)	0.834 (0.000)	0.834 (0.000)
Monthly Data 1983:06 To 1999:09	1983:06 To 1999:10	1983:06 To 1999:11	1983:06 To 1999:12
Usable Observations	197	198	199
Degrees of Freedom	195	196	197
Adjusted R^2	0.700	0.699	0.699
Residual Standard Deviation	0.102	0.101	0.101
Durbin-Watson Statistic	1.878	1.875	1.878
Q(36-1)	21.279 (0.967)	21.383 (0.966)	21.583 (0.963)

[a]P-value.

[b]No forecasts are generated.

Table 9 Out-of-sample, Updated Forecasts of r_t:[a] Extended Sample Period and Shortened Forecast Period

Re-estimation:[b]	None[c]	First[d]	Second	Third	Fourth	Fifth
1999:07	4.9800[e]					
	4.9867[f]					
1999:08	5.0149	5.0206				
	5.0726					
1999:09	5.0440	5.0489	5.0929			
	5.0411					
1999:10	5.0684	5.0726	5.1098	5.0661		
	5.0142					
1999:11	5.0888	5.0923	5.1239	5.0871	5.0432	
	5.1373					
1999:12	5.1058	5.1089	5.1356	5.1045	5.0675	5.1468
	5.1630					

[a]Generated by THEIL instruction in Doan (*RATS*). The sixth (final) re-estimation generates no forecasts.

[b]For the estimation results, see Table 8.

[c]Meaning no re-estimation. Sample Period is From 1983:06 To 1999:06, and Forecast Period From 1999:07 To 1999:12. The forecasts here are exactly the same as those reported in Table 7.

[d]Sample Period is From 1983:06 To 1999:07, and Forecast Period From 1999:08 To 1999:12. The sample and forecast periods are similarly varied for the remaining, second through fifth re-estimations.

[e]The upper figure is a forecast value.

[f]The lower figure is an actual value.

Table 10 Forecast Statistics

Step	Mean Error	Mean Abs Error	RMS Error	Theil U	N.Obs
For r_t: AR[1] Model (2)[a]					
1	0.011	**0.045**[b]	**0.054**	0.800	6
2	0.020	**0.061**	**0.070**	0.658	5
3	0.003	0.033	0.042	0.480	4
4	0.006	0.042	0.044	0.395	3
5	*0.051*	0.051	0.051	*0.274*	2
6	*0.057*	0.057	0.057	*0.255*	1
For r_t^{CPI}: Random Walk Model (9)					
1	-0.025	**0.028**[c]	**0.031**	0.972	6
2	-0.052	**0.052**	**0.058**	0.966	5
3	-0.076	0.076	0.081	0.959	4
4	-0.100	0.100	0.105	0.950	3
5	-0.124	0.124	0.127	0.938	2
6	-0.139	0.139	0.139	0.921	1
For r_t: Random Walk Model (8)					
1	0.040	0.058	0.069	1.019	6
2	0.079	0.101	0.109	1.032	5
3	0.086	0.086	0.093	1.069	4
4	0.115	0.115	0.119	1.078	3
5	0.200	0.200	0.200	1.065	2
6	0.240	0.240	0.240	1.069	1

[a]Some remarks are in order on how to compute the forecast statistics: the italic figures are computed as follows (the underlined figures below are those in Table 9):

- Mean Error (ME):

6-step-ahead ME=average of one 6-step-ahead forecast error (=actual $-\underline{\text{forecast}}$)
$=$*0.057*$=\frac{1}{1}\{5.1630 - \underline{5.1058}\}^{*}$;
5-step-ahead ME=average of two 5-step-ahead forecast errors=*-0.051*
$=\frac{1}{2}[\{5.1373 - \underline{5.0888}\}^{**} + \{5.1630 - \underline{5.1089}\}^{***}]$.

- RMS Error (RMSE):

6-step-ahead RMSE=square root of average of one 6-step-ahead forecast error squared $=\sqrt{\frac{1}{1}(*)^2}$;
5-step-ahead RMSE=square root of average of two 5-step-ahead forecast errors squared$=\sqrt{\frac{1}{2}\{(**)^2 + (***)^2\}}$.

- Theil U:

Theil U at Step ℓ =[RMSE at Step ℓ]/[RMSE at Step ℓ of a naive model, where "naive" means *no-change* (i.e., *flat*) prediction in the sense that the ℓ step ahead forecast is set equal to the actual value at the forecast origin; how to compute italic Theil U's in the table, following Doan (*RM*, pp.420-425), is detailed in Table 11.

[b]Boldface figures are greater than those in the middle panel.

[c]Boldface figures are smaller than those in the top panel.

Table 11 Details of Computing Theil's U Statistic[a]

Step ℓ	N_ℓ=N. Obs	Theil's U_ℓ[b]
6	1	$SSE_{NCF6} = \sum_{i=1}^{N_6}(y_{i6} - \underline{y_{i0}}^{c})^2 = (y_{16} - \underline{y_{10}})^2$, where the subscript 0 indicates a forecast origin, and $SSE_{NCF\ell}$ =SSE of $\underline{\ell\ \text{step ahead flat forecasts}\ (NCF\ell),\ y_{i0}}$, of $y_{i\ell}$ $= (r^{\dagger}_{1999:12} - \underline{r_{1999:6}})^2$, with $y_{16} = r_{1999:12}$ and $y_{10} = r_{1999:6}$ $= \{5.163^{\dagger} - \underline{4.938}\}^2$=0.051; $RMSE_{NCF6} = \sqrt{SSE_{NCF6}/N_6} = \sqrt{0.051/1}$ =0.226 where $RMSE_{NCF\ell}$ =RMSE of $NCF\ell$, and $TheilU_6 = RMSE_6/RMSE_{NCF6}$ where $RMSE_6$ is from Table 10 =0.057/0.226 =*0.252* with some rounding error.
5	2	$SSE_{NCF5} = \sum_{i=1}^{N_5}(y_{i5} - \underline{y_{i0}})^2$ $= (y_{15} - \underline{y_{10}})^2 + (y_{25} - \underline{y_{20}})^2$ $= (r^{\ddagger}_{1999:11} - \underline{r_{1999:6}})^2 + (r^{\dagger}_{1999:12} - \underline{r_{1999:7}})^2$ $= \{5.137^{\ddagger} - \underline{4.938}\}^2$ $+\{5.163^{\dagger} - \underline{4.9867}\}^2$ =0.071; $RMSE_{NCF5} = \sqrt{SSE_{NCF5}/N_5} = \sqrt{0.071/2}$ =0.188; $TheilU_5 = RMSE_5/RMSE_{NCF5}$ = 0.051/0.188 =*0.271* with some rounding error.
4 through 1	3 through 6	Computed similarly as above.

[a]Detailed here is how to compute those italic Theil U's in the top panel of Table 10, following Doan (*RM*, pp.420-425).

[b]Those figures underlined and with a superscript are the following (actual) r_t figures underlined and with a superscript: 1999:06=$\underline{4.938}$, 1999:07=$\underline{4.9867}$, 1999:11= $5.137^{\ddagger}$, 1999:12=$5.163^{\dagger}$.

[c]Naive (i.e., no-change or flat) prediction is set in such a way that the ℓ step ahead forecast is set equal to the actual value at the forecast origin. See Doan (*RM*, pp.423-425).

CPI-based real exchange rate It is remarked in section 3.1 that the unit-root null is *not* rejected for the CPI-based real exchange rate, r_t^{CPI} (plotted in Figure 14).[27] This is widely documented in the existing literature and consistent with the extremely slowly decaying (left) SACF in Figure 18. We thus next forecast the level of exchange rate, r_t^{CPI}, by estimating:

$$r_t^{CPI} = c + \phi_1 r_{t-1}^{CPI} + u_t \tag{9}$$

where the estimate of ϕ_1 should not be statistically different from unity, and c is a possibly non-zero drift term. Table 12 provides the estimated results (the estimated ϕ_1 of 0.977, in particular) consistent with both the unit-root test result (in the middle panel of Table 3) and the extremely gradually decaying SACF (in Figure 18). The dynamic out-of-sample forecasts are tabulated in Table 13 and plotted in Figure 21.

The forecasting performance statistics are shown in the middle panel of Table 10.[28] The Theil's U statistics (of between 0.921 and 0.972) in the middle panel suggest that the random walk model appears to perform almost as well as the naive (flat) forecasting model, for all the months ahead; this is in line with our expectation on the nature of a random walk process but in contrast with performance of the stationary r_t model (2).

4.3.2 Forecasting performance comparison: r_t versus r_t^{CPI}

Comparing the forecasting performance statistics compiled in the top and middle panels of Table 10, we see that the (stationary) AR[1] model for r_t performs better than the random walk model for r_t^{CPI}, except that, based on Mean Absolute Error and RMS Error statistics, the random walk model outperforms the AR[1] model at horizons as short as one and two months (as is clear from boldface figures in the two tables). The results may be interpreted as follows:

Medium/long run The forecasting performance result for three- to six-month forecast horizons indeed coincides with our earlier intuitive

[27] See the middle panel of Table 3.

[28] Details of computing the forecast statistics here are omitted; note that, as in the extracted price index-based real exchange rate model, the forecasting performance statistics are computed based on a set of updated forecasts which are different from those in Table 13.

Table 12 Box-Jenkins - Estimation by Gauss-Newton:[a] Model (9)

Dependent Variable	r_t^{CPI}
Constant	5.020 (0.000)[b]
r_{t-1}^{CPI}	0.977 (0.000)
Monthly Data	1983:06 To 1999:06
Usable Observations	193
Degrees of Freedom	191
Adjusted R^2	0.970
Residual Standard Deviation	0.036
Regression F(1,191)	6269.601 (0.000)
Durbin-Watson Statistic	1.886
Q(36-0)	24.609 (0.949)

[a]Convergence in 3 Iterations. Final criterion was 0.0000000 < 0.0000100

[b]P-value.

Table 13 Out-of-sample Forecasts of r_t^{CPI}:[a] Forecast Period From 1999:07 To 1999:12

1999:07	5.111
1999:08	5.108
1999:09	5.106
1999:10	5.104
1999:11	5.102
1999:12	5.101

[a]The (dynamic) forecasts are computed by the estimated model in Table 12.

conjecture that, in the long run, stationary series would be less difficult to predict than nonstationary (random-walk) time series. As our conjecture is thus supported by the data for medium/long run (such as over three- to six-month forecast horizons), earlier evidence (in section 3) of a long-run, mean-reverting nature of r_t would be here reinforced.

Short run On the other hand, in the short run (such as over one- to two-month forecast horizons), the random walk model is found to perform better. This seems consistent with both (i) a tendency of random-walk r_t^{CPI} to move in a certain direction for some (short) period of time,[29] and (ii) the short-run erratic movement of stationary r_t: see "Actual" in Figure 21 for the tendency of r_t^{CPI} in a certain direction for some (short) period of time and contrast it with the erratic behavior of r_t drawn as "Actual" in Figure 20.

[29]The tendency is captured well by the non-zero drift term, $c = 5.020$, in Table 12.

Overall, the out-of-sample forecasting performance study here adds evidence consistent with a stationary nature of r_t and a random-walk nature of r_t^{CPI}.

4.3.3 Forecasting performance comparison: stationary AR[1] versus random walk, for r_t

One remark is in order on another question raised earlier: which model predicts out-of-sample r_t better, AR[1] model (2) or random walk model (8). The forecasting performance statistics are computed for the random walk model and reported in the bottom panel of Table 10: comparing them with those for the AR[1] model in the top panel of the table, we readily see that the stationary AR[1] model outperforms the random walk model, for *all* the months ahead. This, too, confirms the stationary (long-run, mean-reverting) nature of r_t in the context of the out-of-sample forecasting performance.

5 Concluding Remarks

The paper documents evidence supportive of a stationary nature of extracted price index-based Japanese yen per U.S. dollar real exchange rate, r_t. The price inflation data used is a set of the estimates for realized pure price inflation rates extracted from stock returns by Chowdhry, Roll and Xia (2005). Employing three methods of modeling, it is shown consistently that r_t obeys a long-run, mean-reverting stationary process. The mean-reverting behavior detected is consistent with the less restrictive version of absolute PPP in which a real exchange rate is allowed to temporarily deviate from its mean. The stationarity result is reinforced, in particular, by the out-of-sample forecasting performance comparison between r_t and the random-walk CPI-based real exchange rate, r_t^{CPI}: in the long run, stationary mean-reverting r_t appears less difficult to predict than nonstationary r_t^{CPI}.

These major findings include the following further evidence:

Based on the summary statistics and graphical inspection, the extracted inflation rate differentials between Japan and the U.S. fluctuate much more highly than the CPI inflation rate differentials, implying that r_t should behave very differently from s_t, and hence from r_t^{CPI}. We note that, while extracted price index and CPI are cointegrated, r_t and r_t^{CPI} computed based on them exhibit sharply differing behavior due to distinct difference in volatility between extracted and CPI inflation rate differentials.

Modeling real exchange rate, r_t, we find the following consistent results: (i) the unit-root testing of r_t itself leads to decisive rejection of the null of a random walk; (ii) exploiting the Engle-Granger (1987) two-step cointegration methodology, s_t, p_t^* and p_t are shown to comove in the long run with one another, which eventually implies that r_t defined as $s_t + p_t^* - p_t$ would not be nonstationary; and (iii) the Box-Jenkins (1976, 1994) model identification technique leads to a first-order AR[1] process for which we infer that its estimated AR parameter is less than unity in absolute value (and hence the stationarity condition is satisfied), on grounds of the two earlier strong test results (i) and (ii).

Implications of stationary real exchange rate are presented with respect to absolute PPP, equilibrium error, expectations about future, and out-of-sample forecasting performance. They are found to help shed light on the stationary features of the real exchange rate, r_t. We claim that the stationarity of r_t is fundamentally attributable to frequent and sharp changes in expectations reflected in goods prices that are implied by the extracted pure price inflation rates. In the out-of-sample forecasting performance study, the data support our intuitive conjecture that, in the long run (such as over three- to six-month forecast horizons), stationary series (r_t) would be less difficult to predict than nonstationary (random-walk) time series (r_t^{CPI}), because of the long-run, mean-reverting behavior of r_t. On the other hand, in the short run (such as over one- to two-month forecast horizons), the random walk model for r_t^{CPI} is found to perform better; the result seems consistent with both a tendency of random-walk r_t^{CPI} to move in a certain direction just for a short period of time, and the short-run erratic movement of stationary r_t. Further, in forecasting out-of-sample r_t, stationary AR[1] model outperforms random walk model; this, too, confirms the stationary (long-run, mean-reverting) nature of r_t.

A similar study may be conducted using the U.K. and German inflation rates extracted from the assoicated stock returns by C-R-X. The topics explored for Japan deserve further research in an attempt to provide multi-country evidence on the stochastic nature of real exchange rates.

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