Time Series Analysis of the Japanese Yen, the Euro and the Chinese Yuan Exchange Rates
Univariate and Multivariate Evidence from Daily Data

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Abstract

This paper studies the individual and joint behavior of three daily exchange rates (the Japanese Yen, the Euro and the Chinese Yuan), all against a U.S. dollar, during the period of June 21, 2010 - December 30, 2016. First, logged and then first-differenced, the Japaenese Yen is found to behave according to either AR(19) or MA(19), while the Euro a white noise. Second, the Yuan requires an intervention analysis/model incorporating two permanent level shifts invoked by the Chinese central bank's decision of huge devaluation. Detected were over 1.8% devaluation on August 11 and nearly 1% devaluation on August 12, both in 2015. Third, for the period of June 21, 2010 - August 10, 2015, the longest period of time when the Yuan was continuously less managed or controlled by China's central bank under (managed) flexible exchange rate system, a posteriori the VAR modeling detects no cointegration relationships among the three daily exchange rates, and yet the chi-squared tests for their unrestricted VAR models (that is, VAR models with no a priori restrictions/cointegrations) show that even the China's Yuan exchange rate controlled carefully by the Chinese central bank and government enters into the picture as a statistically significant constituent of the multivariate daily exchange-rate model.

*Department of Commerce, Seinan Gakuin University, Fukuoka, Japan. E-mail: kojima@seinan-gu.ac.jp   A business time-series forecasting system was built by my past study Kojima(2005), with the intervention analysis of Japanese Yen exchange rate behavior. In more recent years (from mid 2010 through present, in particular) the Chinese currency's exchange rate has been continuously managed or controlled by China's central bank under (managed) flexible exchange rate system. Together these two motivated me to initiate the present (univariate and multivariate) time series econometric research.

1   Introduction

A priori (*deductively*) no set of exchange rates is expected to have a cointegration relationship: No theory postulates that exchange rates are cointegrated.[1] Yet *a posteriori* (*inductively*) exploring for the evidence of a cointegration relationship is a worthwhile empirical/data-driven research, and the two-fold purpose of the present paper is to individually study the behavior of three daily exchange rates (the Japanese Yen, the Euro and the Chinese Yuan), all against a U.S. dollar, in a univariate time series framework, and further to research the joint behavior of the three exchange rates by a multivariate time series model.

The Chinese Yuan's exchange rate is selected and studied primarily because China has adopted varied systems of exchange rate in the past past quarter of a century including a flexible, though managed, system, as is summarized in Tables 1 and 2. The two tables, together with Figs. 1 through 7, document that the Chinese Yuan (CNY/USD) was less controlled or managed by China's central bank and government during the two periods III and V (July 21, 2005 - July 31, 2008 and June 21, 2010 - August 10, 2015) than during the remaining periods.[2]

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[1]Meanwhile, an a priori cointegration relationship is postulated in the context of purchasing power parity (PPP), as briefly summarized in Kojima (2006a, pp.5-6): "To be specific, the paper considers a system of three economic variables (all logged): a nominal exchange rate s_t of a home currency (Japanese yen) against a foreign currency (U.S. dollar), a foreign price index p_t^* and a home price index p_t, where the price indices are constructed from Chowdhry, Roll and Xia's (2005) (C-R-X's) extracted inflation rates. ...";

"(W)ith the real exchange rate defined as a particular linear combination $r_t \equiv s_t + p_t^* - p_t$, absolute PPP asserts that $s_t + p_t^* = p_t$, that is, $r_t = 0$. *[A footnote is attached here in the original paper that (r)elative PPP requires, in terms of percentage, that* $\Delta s_t + \Delta p_t^* = \Delta p_t$ *where* Δ *is the first difference operator.]* Based on the preliminary results obtained, we will next carry out a formal cointegration analysis of the entire three series (s_t, p_t^* and p_t). ...

Second, are the entire three series (s_t, p_t^* and p_t) cointegrated? In the cointegration analysis of the three series, a particular restriction motivated by our economic arguments will be the (strong) PPP restriction (1   1 -1), which is in fact a vector of coefficients on the right-hand side of the real exchange rate's definition above."

The present paper only looks at $s_{it}, i =$ Japanese Yen, Euro and Chinese Yuan, which are not a priori expected to be cointegrated.

[2]During the period I (January 3, 1994 - December 31, 1996) there occurred huge devaluation and appreciation (as controlled by the Chinese central bank), respectively, on Monday, December 19 and Tuesday, December 20, 1994; during the period V through 2016 (June 21, 2010 - December 30, 2016) there occurred huge devaluation two days in a row (again, as controlled by the Chinese central bank), respectively, on Tuesday, August 11 and Wednesday, August 12, 2015; and in the remaining years

Table 1 Exchange Rate Systems in China since 1994, along with Variability of Daily Rate of Change in CNY/USD (GRCNYUSD): Panel 1[a]

I. Monday, January 3, 1994 - Tuesday, December 31, 1996 $(T = 767$ for Raw, Undifferenced Data):[b] *Flexible (Essentially, Pegged-to-U.S. Dollar) Exchange Rate System.*			

Statistics on Series GRCNYUSD

Observations 745[c] Skipped/Missing 21[d]

Sample Mean	-0.000060	Variance	0.000002[e]
Minimum	-0.021362[f]	Maximum	0.020855[g]
Median	-0.000048		

II. 1997 - Wednesday, July 20, 2005: *Fixed Exchange Rate System.*

III. Thursday, July 21, 2005 - Thursday, July 31, 2008 $(T = 761$ for Raw, Undifferenced Data):[h] *Managed Flexible Exchange Rate System.*

Statistics on Series GRCNYUSD

Observations 760[i]

Sample Mean	-0.000225	Variance	0.000001[j]
Minimum	-0.004599	Maximum	0.003163[k]
Median	-0.000169		

IV. August 2008 - Friday, June 18, 2010: *Fixed Exchange Rate System.*

[a]See Fig. 1 or 2. Source: BJidentify_fixdata_Jan31994-Dec311996output, BJidentify_fixdata_Jul212005-Jul312008output and BJidentify_fxdataoutput.

[b]This is the shaded period without vertical grid lines in Fig. 1 and 2. See Fig. 4 for DLOGCNYUSD that closely approximates GRCNYUSD.

[c]This equals $T' - Missing = T - d - Missing = 767 - 1 - 21$ where T' denotes the effective sample size (the number of differenced data) and d the order of (consecutive) differencing required to compute the rate of change GRCNYUSD; see Table 3 in Subsection 3.1 for the notation.

[d]This is due to the dates when (raw) JPYUSD is available but not CNYUSD and EURUSD: They are 11th, 36th, 65th, 66th, 106th, 131st, 231st, 266th, 291st, 361st and 387th dates; and thus daily rates of change are not available at twenty one dates (11th, 12th, 36th, 37th, 65th, 66th, 67th, 106th, 107th, 131st, 132nd, 231st, 232nd, 266th, 267th, 291st, 292nd, 361st, 362nd, 387th and 388th dates). For details see the very first output in the source list.

[e]An unbiased sample variance (Doan 2007a, p.441). The sample standard deviation =%sqrt(%variance) = 0.00138.

[f]Huge appreciation on Tuesday, December 20, 1994 (249th date).

[g]Huge devaluation on Monday, December 19, 1994 (248th date).

[h]This is the shaded period without vertical grid lines in Fig. 1 and 2.

[i]Friday, July 22, 2005 - Thursday, July 31, 2008.

[j]An unbiased sample variance (Doan 2007a, p.441). The sample standard deviation =%sqrt(%variance) = 9.56183e-04 or 0.000956183.

[k]Range (=maximum-minimum)=0.00776.

Table 2   Exchange Rate Systems in China since 1994, along with Variability of Daily Rate of Change in CNY/USD (GRCNYUSD): Panel 2[a]

V. Monday, June 21, 2010 - Monday, August 10, 2015 ($T = 1286$ for Raw, Undifferenced Data):[b] *(Managed) Flexible Exchange Rate System* (Continuing through Present).			
Statistics on Series GRCNYUSD			
Observations 1285[c] Skipped/Missing 1			
Sample Mean	-0.000070	Variance	0.000001[d]
Minimum	-0.005912	Maximum	0.006042[e]
Median	-0.000075		

 [a]See Fig. 1 or 2. Source: BJidentify_fixdata_Jan31994-Dec311996output, BJidentify_fixdata_Jul212005-Jul312008output and BJidentify_fxdataoutput.
 [b]This is the shaded period with vertical grid lines in Fig. 1 and 2.
 [c]Tuesday, June 22, 2010 - Monday, August 10, 2015.
 [d]The (unbiased) sample standard deviation =%sqrt(%variance) = 0.00114, which is larger than that for the period of Friday, July 22, 2005 - Thursday, July 31, 2008 above (see footnote j of Table 1).
 [e]Range (=maximum-minimum)=0.01195, which is larger than that for the period of Friday, July 22, 2005 - Thursday, July 31, 2008 above (see footnote k of Table 1).

 Further, footnotes d and e in particular of Table 2 evidence that in terms of both sample standard deviation and range (=maximum-minimum) the exchange rate variability turns out larger during the latter period V (June 21, 2010 - August 10, 2015), implying that during the latter period CNY/USD turns out more flexible or volatile (whether management/control by the Chinese central bank is aggressive or not).[3]

 The three daily exchange rate data are all extracted from the Database Retrieval System (v2.11), available at the University of British Columbia's Sauder School of Business (http://fx.sauder.ubc.ca/data.html). The sample period is period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016) [$T = 1634$ Observations], for the univariate time series analysis of daily exchange rates, whereas it is period V (Monday,

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CNY/USD is kept nearly constant under the fixed exchange rate system adopted.
 [3]Recall from the immediately preceding footnote that both on August 11 and 12, 2015, the central bank in China unanticipatedly and heavily controlled the Yuan by significantly devaluing it; this naturally increased its variability during the period extending beyond August 10 (see Figs. 5 through 7). The spikes in CNY/USD on the two dates will be studied in the framework of (univariate) intervention model in Section 4.

June 21, 2010 - Monday, August 10, 2015) $[T = 1286$ Observations] for the VAR modeling of the three daily exchange rates. For the latter sample period see a note on the shaded period with vertical grid lines in Figs. 1 through 6.

The sample periods are so chosen, in part since the Yuan requires an intervention analysis/model incorporating two permanent level shifts invoked by the Chinese central bank's decision of huge devaluation two days in a row in mid-August 2015 (see the two preceding footnotes), which is the largest devaluation effected in China's system/Yuan in over 20 years (since 1994).[4]

The paper proceeds as follows: The relevant literature is reviewed in Section 2. Univariate time series models for JPY/USD and EUR/USD, period V through 2016, are identified and estimated in Section 3; a univariate intervention model for CNY/USD (incorporating two permanent level shifts) is next identified and estimated in Section 4. Section 5 attempts to build VAR models to study the joint behavior of the daily JPY/USD, EUR/USD and CNY/USD, period V, by conducting cointegration tests and (F and chi-squared) tests on three differing nulls of lagged regressor(s) being excluded/omitted. Several concluding remarks are made in the final section. Two appendices follow: Figure appendix contains figures drawn for (i) univariate time series analysis of the the exchange rates and (ii) VAR modeling; table appendix tabulates the source of each table, and test results for (i) testing the individual series for unit roots, (ii) single-equation based, two-stage test of cointegration, (iii) (multivariate) VAR based likelihood ratio test of cointegration, and (iv) setting the lag length of the unrestricted VAR model.

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[4]See Table 1 evidencing huge devaluation ("Maximum = 0.020855") and appreciation ("Minimum = -0.021362"), two days in a row, that is, respectively, on Monday, December 19, 1994 and Tuesday, December 20, 1994. See also Fig. 4: As is readily clear from the figure, variation of such magnitude occurs quite frequently in the Japanese Yen in particular.
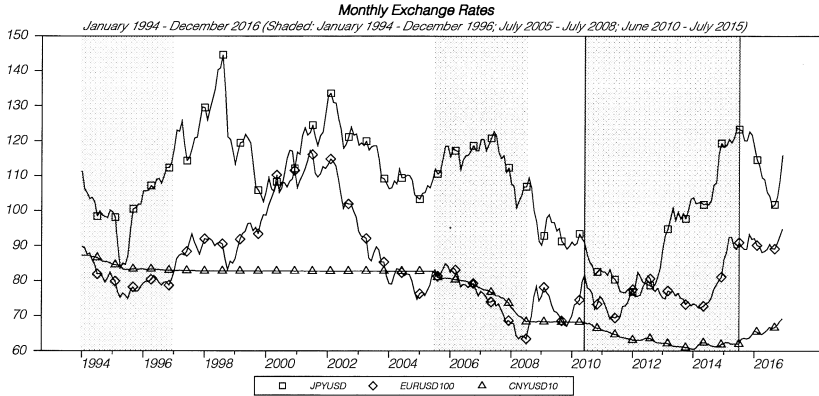
Figure 1 Monthly Exchange Rates, January 1994 - December 2016 (Shaded: January 1994 - December 1996; July 2005 - July 2008; June 2010 - July 2015 with vertical grid lines). Note 1: Drawn for a clear exposition are EURUSD100(=EUR/USD×100) and CNYUSD10(=CNY/USD×10). Note 2: The shaded period with vertical grid lines is the longest period of time when CNY/USD was continuously less managed/controlled by the central bank in China under (managed) flexible exchange rate system (see Section 1 referring to Tables 1 and 2); this applies to Figs. 2 through 6 as well.
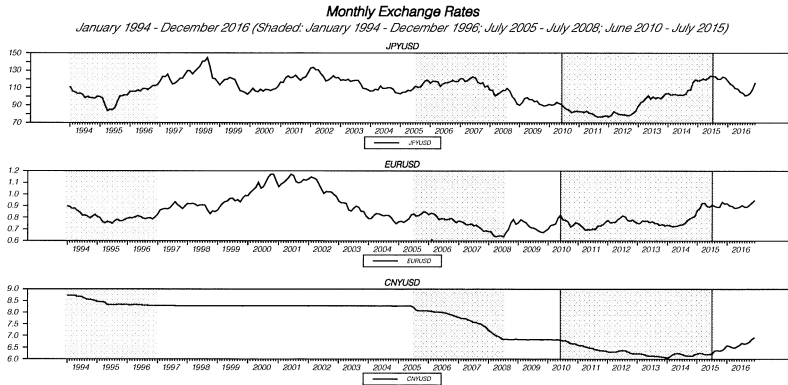

Figure 2 Monthly Exchange Rates, January 1994 - December 2016 (Shaded: January 1994 - December 1996; July 2005 - July 2008; June 2010 - July 2015 with vertical grid lines). Note: See Note 2 in Fig. 1 for the shaded periods.
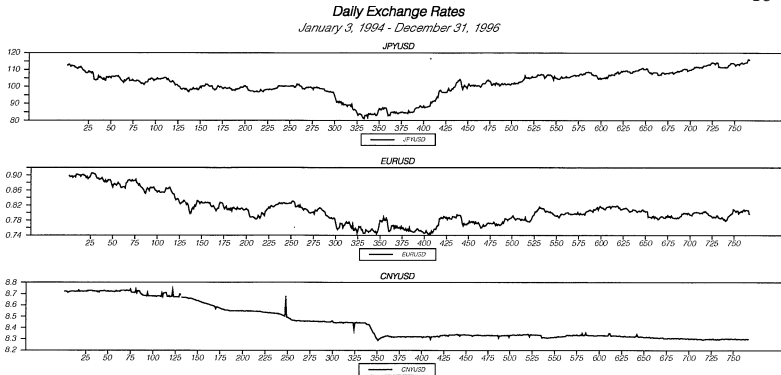
Daily Exchange Rates
January 3, 1994 - December 31, 1996

Figure 3   Daily Exchange Rates, Period I (Monday, January 3, 1994 - Tuesday, December 31, 1996) $[T = 767$ Observations]. Note: One spike observed in CNYUSD at 248th date, an additive outlier (AO), corresponds to huge devaluation ("Maximum = 0.020855" in Table 1) on Monday, December 19, 1994. See Subsection 4.3.1 for AO (as compared to permanent level shifts (PSs) that occurred on August 11 and 12, 2015, as drawn in Fig. 6).
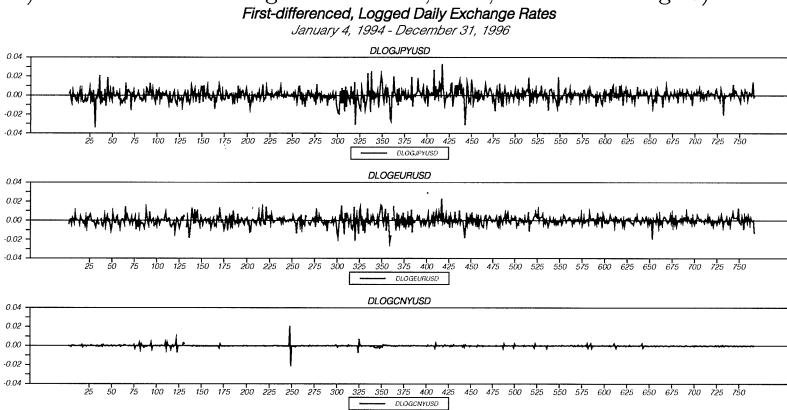
First-differenced, Logged Daily Exchange Rates
January 4, 1994 - December 31, 1996

Figure 4   First-differenced, Logged Daily Exchange Rates, Tuesday, January 4, 1994 - Tuesday, December 31, 1996 $[1 + d$ to $T : 2$ to 767, with $T' = T - d = 767 - 1$ where T' denotes the effective sample size (the number of differenced data) and d the order of (consecutive) differencing required to compute DLOGCNYUSD; see Table 3 in Subsection 3.1 for the notation]. Note: Two spikes observed at 248th date and 249th date correspond to huge devaluation ("Maximum = 0.020855" in Table 1) and appreciation ("Minimum = -0.021362" in Table 1), two days in a row, that is, respectively, on Monday, December 19, 1994 and Tuesday, December 20, 1994.
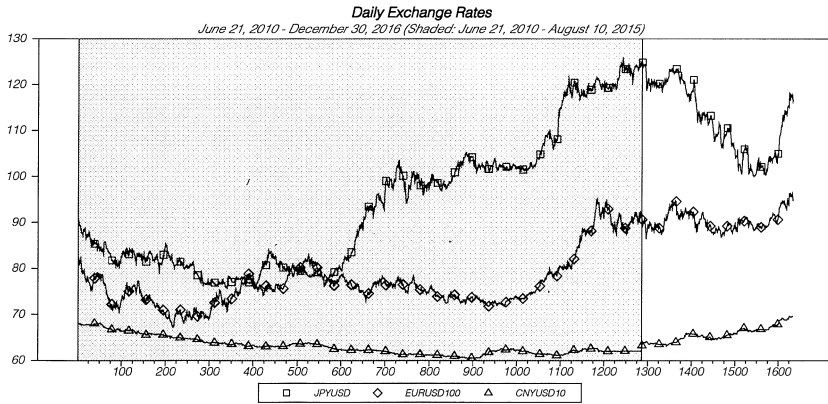
Figure 5 Daily Exchange Rates, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016) [$T = 1634$ Observations] [Shaded: Period V (Monday, June 21, 2010 - Monday, August 10, 2015); $T = 1286$ Observations]. Note: See Note 1 and, for the shaded period with vertical grid lines, Note 2, in Fig. 1.
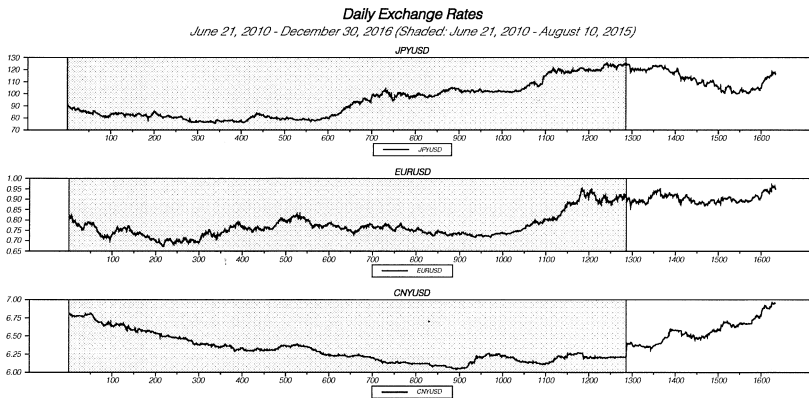

Figure 6 Daily Exchange Rates, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016) [$T = 1634$ Observations] [Shaded: Period V (Monday, June 21, 2010 - Monday, August 10, 2015); $T = 1286$ Observations]. Note: See Note 2 in Fig. 1 for the shaded period with vertical grid lines.
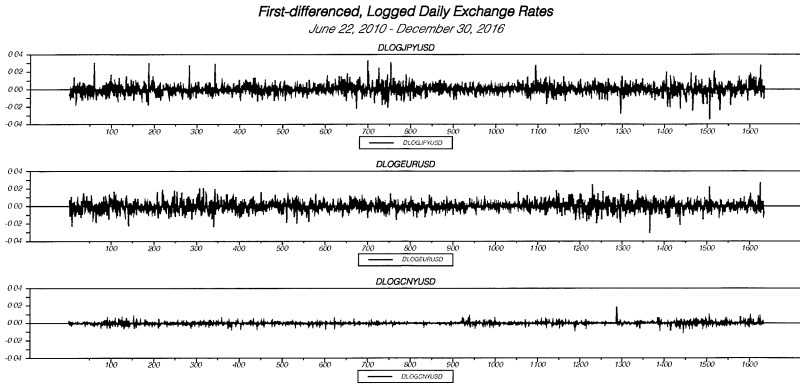
First-differenced, Logged Daily Exchange Rates
June 22, 2010 - December 30, 2016

Figure 7 First-differenced, Logged Daily Exchange Rates, Tuesday, June 22, 2010 - Friday, December 30, 2016 $[1 + d$ to $T : 2$ to 1634, with $T' = T - d = 1634 - 1$; see Fig. 4 for the notation here]. Note 1: Unlike Fig. 6, the period V (Monday, June 21, 2010 - Monday, August 10, 2015) is not shaded so as to highlight the spikes in DLOGCNYUSD on Tuesday, August 11 and Wednesday, August 12, 2015 (=1287th. and 1288th. observations, with Tuesday, June 22, 2010 = 2nd. observation); for the two spikes see Table 12 in Subsection 4.3.2, too. Note 2: See Subsection 3.1 for the economic interpretation of first-differenced, logged series as a rate of change.

Daily Exchange Rates
June 21, 2010 - August 10, 2015 (Excluding August 11 on, for Cointegration Tests)
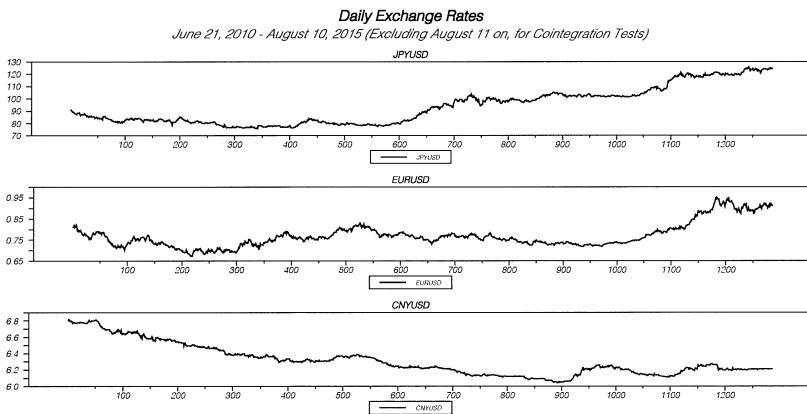

Figure 8 Daily Exchange Rates, Period V (Monday, June 21, 2010 - Monday, August 10, 2015) (Excluding August 11 On, for VAR Model) $[T = 1286$ Observations]. Note: The daily sample period here is the shaded one with vertical grid lines, as noted in Figs. 5 and 6.

2   Literature Review

The past, fundamental literature includes Box and Jenkins (1976) for univariate time series analysis, Dickey and Fuller (1979) and Phillips (1987, 1988) for a univariate testing for a unit root, and Johansen (1988, 1991) for (multivariate) VAR based, likelihood ratio test of cointegration.

   Building upon them, Kojima (1993, 1994, 2005, 2006a, 2006b, 2010), dating back over a quarter of a century, constitute a foundation of the present research, but did not attempt to statistically explore for, and specify, a joint behavioral pattern of a multiple exchange rates. Bridging the gap, thus, the present paper aims to newly study the joint behavior of the three exchange rates above, in particular by adding China's Yuan which has been experiencing varied systems of exchange rate over twenty years (as shown in Table 2), while Japanese Yen and Euro have been continuously under flexible exchange rate system. One question of interest, which has not yet been investigated, is then whether or not the Chinese currency's exchange rate will be a statistically significant component of the joint behavioral pattern or system.

3   Identification and Estimation: JPY/USD and EUR/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016)

3.1   Identification: JPY/USD and EUR/USD

Univariate time series models for exchange rates are identified based on Kojima (2005, pp.43-49), which is quoted below ("...") by refering to the original section/appendix numbers, table numbers and figure numbers but having the equation numbers and footnote numbers changed:

   "(Univariate t)ime-series models will be generally identified through two phases:

   The first-half phase: The stationarity of the raw data X_t will be examined: if it is found nonstationary, then some work will be required to transform it into stationary series W_t. The stationarity conditions are that neither the expected value of W_t nor the population autocovariance

$Cov(W_t, W_{t-l}), l \geq 1$ depends on t:

$$E[W_t] = \mu; \tag{1}$$

$$\sigma_{W_t, W_{t-l}} = \gamma_l. \tag{2}$$

The second-half phase: Time-series model(s) that suit well the data at hand will be selected as candidate(s). Again, those selected models must satisfy both usual stationarity and invertibility conditions.[5]

Time-series models for the raw data X_t to be considered in this phase are multiplicative SARIMA$(p, d, q; P, D, s, Q)$ models (seasonal ARIMA models): X_t is assumed to have not only trend but seasonal variation. Its $(d; D, s)$-th differenced series

$$W_t = (1 - B)^d (1 - B^s)^D X_t, \tag{3}$$

where d denotes a consecutive difference order and D a seasonal difference order, is assumed to satisfy stationarity conditions (1) and (2). With T denoting the sample size (the end of the sample period), the effective sample size (the effective end of the differenced series) is $T' = T - d - sD$.[6]

Let now

$$X_t^\ell = \log X_t, \tag{4}$$

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[5] See Box, Jenkins and Reinsel (1994, pp.50-51) for invertibility conditions: "To sum up, a linear process $z_t - \mu_z = a_t + \sum_{j=1}^{\infty} \psi_j a_{t-j}$, with a_t being a white noise, is stationary if $\sum_{j=0}^{\infty} |\psi_j| < \infty$ and is invertible if $\sum_{j=0}^{\infty} |\pi_j| < \infty$, where $\pi(B) = \psi^{-1}(B) = 1 - \sum_{j=1}^{\infty} \pi_j B^j$." For π see also Appendix A.

[6] *This footnote is newly added for the present paper: More specifically, $t = 1, \cdots, T'$ for the differenced data corresponds to $1 + d + sD, \cdots, T$ associated with the raw (undifferenced) data series. Note further that, for the <u>residuals</u> series (the estimated a_t) for Eq. (5) below, $T'^r = T' - \max\{p, sP\}$ so that its $t = 1, \cdots, T'$ corresponds to $1 + d + sD + \max\{p, sP\}, \cdots, T$ associated with the raw (undifferenced) data series.*

This will be illustrated by (i) the <u>residuals</u> of the VAR model for which $d + sD = 0$ (and thus $T' = T$ and $T'^r = T - \max\{p, sP\}$), as noted in Table 13 and immediately below Eq. (40) in Subsection 5.3.3, and (ii) "Usable Observations" and "Observations (for Statistics on Series <u>RESIDS</u>)" (both in RATS programming) in Table 10 in Subsection 4.3.2.

In sum, see Table 3 (newly constructed for the present paper) in the text: $t = 1, \cdots, T'$ for the the differenced data series corresponds to $1 + d + sD, \cdots, T$ associated with the raw (undifferenced) data series; $t = 1, \cdots, T'^r$ for the <u>residuals</u> corresponds to $1 + d + sD + \max\{p, sP\}, \cdots, T$ associated with the raw (undifferenced) data series. Every RATS program (for example, VAR_VECM_fxdata.prg witten to construct Tables 13 and 14 in Subsection 5.3.3) incorporates this technicality (as summarized in Table 3).

Table 3   Time Framework for Raw (Undifferenced) Data, Differenced Data and Reisduals Series

Raw (Undifferenced) Data X_t	Differenced Data W_t	Reisduals Series $e_t{}^a$
1		
2		
$\vdots$		
$1 + d + sD$	1	
$\vdots$	$\vdots$	
$1 + d + sD + \max\{p, sP\}$	$1 + \max\{p, sP\}$	1
$\vdots$	$\vdots$	$\vdots$
T	$T'(= T - d - sD)$	$T'^r(= T' - \max\{p, sP\})$

^{a}For this notations see Subsection 5.3.3.

with which $W_t = (1 - B)^d (1 - B^s)^D X_t^\ell$ will be interpreted as a rate of change from previous period (for $d = 1$, $D = s = 0$) or a rate of change from same period of previous year (for $d = 0$, $D = 1$, $s = 12$ for monthly data or $s = 4$ for quarterly data).[7] Then, with a_t denoting the white noise, SARIMA$(p, d, q; P, D, s, Q)$ for X_t^ℓ will be written as

$$\phi(B)\Phi(B^s)(1 - B)^d (1 - B^s)^D X_t^\ell = \theta(B)\Theta(B^s)a_t \qquad (5)$$

where $\phi(B), \Phi(B^s), \theta(B), \Theta(B^s)$ are, respectively, AR, SAR, MA, SMA multinomials of backshift operator B, which, with $\phi_0 = \Phi_0 = \theta_0 = \Theta_0 = -1$, are written as:

$$\phi(B) = -\sum_{i=0}^{p} \phi_i B^i; \ \Phi(B^s) = -\sum_{i=0}^{P} \Phi_i B^{is};$$

$$\theta(B) = -\sum_{i=0}^{q} \theta_i B^i; \ \Theta(B^s) = -\sum_{i=0}^{Q} \Theta_i B^{is}. \qquad (6)$$

The multiplicative SARIMA$(p, d, q; P, D, s, Q)$ model (5) may then be

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[7] *This footnote is newly added for the present paper: See Table 12 in Subsection 4.3.2.*

written, too, as:

$$(1 - B)^d (1 - B^s)^D X_t^\ell = c - \sum_{\substack{i=0 \\ Not\{i=0, j=0\}}}^{p} \sum_{j=0}^{P} \phi_i \Phi_j (1 - B)^d (1 - B^s)^D X_{t-i-js}^\ell$$

$$+ \sum_{i=0}^{q} \sum_{j=0}^{Q} \theta_i \Theta_j a_{t-i-js} \tag{7}$$

where: $Not\{i = 0, j = 0\}$ means i and j cannot be both zero simultaneously; the overall constant c is[8]

$$c = \mu \left(1 - \sum_{i=1}^{p} \phi_i \right) \left(1 - \sum_{i=1}^{P} \Phi_i \right)$$

where μ is as given by (1).

Time-series models for W_t to be considered in this phase are multiplicative $SARMA(p, q; P, s, Q)$ models. Denoting the deviation from mean as $\tilde{W}_t = W_t - \mu$, the Multiplicative $SARMA(p, q; P, s, Q)$ model for $\tilde{W}_t$ takes the following form of multiplication of the ϕ, Φ and θ, Θ parameters:

$$\phi(B)\Phi(B^s)\tilde{W}_t = \theta(B)\Theta(B^s)a_t. \tag{8}$$

Sample autocorrelations and partial autocorrelations for SARMA models are simulated and drawn by the RATS program SacfSpacf.prg. Assuming in Eq. (8) that $\mu = 0$ and $\tilde{W}_t = W_t$, SacfSpacf.prg produces, by simulation, Figs. 3 through 7."

Based on Fig. 9 in Appendix A, the first-differenced, logged daily JPY/USD is identified as either an AR(19) or an MA(19); it may suffice to first estimate the former (see the next subsection).

Based on Fig. 11 in Appendix A, the first-differenced, logged daily EUR/USD is appropriately identified as a white noise, to be estimated the next subsection.

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[8]The interpretation of c in the SARIMA model (7) is given as follows: the overall constant c is *included* in the model technically to take into account the possibility that the differenced series W_t in (3) has mean $\mu \neq 0$ (Nelson 1973, p.174), which in turn suggests the presence of upward or downward trend in the *raw, undifferenced* data X_t.

Not including the constant, then, would imply the contrary: W_t has $\mu = 0$ and the raw data has neither type of trend (Nelson 1973, p.63).

3.2   Estimation: JPY/USD and EUR/USD

Univariate time series models for exchange rates are estimated based on Kojima (2005, pp.51-52), which is quoted below ("...") by refering to the original section number but having the footnote numbers changed:

"In the first-half phase of estimation, model(s) identified in section 3 is (are) estimated to compute *initial* estimates, and the stationarity and invertibility conditions of the AR/SAR and MA/SMA estimates, respectively, are checked.[9] With these estimates, one goes on to the second-half phase of an *iterative* model estimation,[10] which is followed by the model diagnostic checking.[11] The models will be re-estimated based on the checking results to further improve their model adequacy.

Those diagnostic checks listed in the previous footnote are carried out later in Outputs 1 and 2 as follows:

For (i) in the footnote, see (A) in Outputs 1 and 2.

For (iii) and (iv), see (B) Check the normality of RESIDS in Outputs 1 and 2.

For (v), see, in Outputs 1 and 2, (C) SCCF Check: A large SCCF at a lag $l < 0$ suggests an AR parameter to be inserted at that l; the parameter value should be close to that SCCF, and (D) SACF Check: A large residuals SACF at a lag l suggests an MA term to be added at l; the parameter value

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[9]See Kojima (1994, Appendix A.1) for details.

[10]See Kojima (1994, pp.11-16) for details.

[11]In the diagnostic checking, the white noise a_t in (5) and (8) is assumed to follow the normal distribution (the white noise normality assumption), under which the residuals distribution and independence are looked into. Here is a list of critical items to be checked:

(i) Is each parameter statistically significant? (ii) Do those final parameter estimates satisfy the stationarity and invertibility conditions? (iii) Is there detected any abnormal behavior in the residuals series? Is the behavior cyclical in nature? (iv) Can the residuals be considered normal? If yes, then one could check on their serial independence by their serial correlations (see the preceding footnote). (v) Would adding a new parameter contribute to improving the model? Or, is there any room for simplifying the model based on the principle of parsimony?

A remark is in oder about residuals distribution and independence. Generally, the necessary and sufficient condition that random variates, like the white noise time series, follow a multivariate normal distribution is that they are uncorrelated (i.e., the covariance matrix of the multivariate distribution is diagonal) (Ferguson 1967, p.110). Note here that the random variates following normal marginal distributions but non-normal multivariate distribution are *not* independent even if they are uncorrelated (Ferguson 1967, p.111).

should be close to the negative of that SACF.[12]"

Throughout the paper, (A) through (D) above will be checked by look-
ing carefully at the tables and figures summarizing the estimation results.
For (B), the number of classes set for a residual histogram is computed
based on the Sturges' rule: fix[1 + log(sample size)/ log(2)] where fix[13]
converts decimal values to integers and log is any logarithmic function
(for example, a natural or an ordinary log); it turns out 11.[14]

3.2.1 First-differenced, logged daily JPY/USD

First, AR(19) with a constant is estimated to find the constant statisti-
cally insignificant at any conventional levels. AR(19) without a constant,
and also MA(19) without a constant, are then estimated to find ϕ_{19} just
slightly more significant than θ_{19}: See Tables 4 and 5 and Figs. 12 and
13 in Appendix A.

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[12]See Hokstad (1983) for these diagnostic checking techniques.

[13]This is a RATS programming terminology (Doan 2007b, p.137).

[14]A simpler computation, $\sqrt{\text{sample size}}$, is not used in the present paper, for this
will result in more than 30 classes for the sample size larger than 1000. (The simpler
one as well as the Sturges' rule both lead to smaller than 10 for the sample size fewer
than 100, however.)

Table 4   Estimated AR(19) Model for First-differenced, Logged Daily JPY/USD: $T = 1634^a$

Variable	Coeff	Std Error	T-Stat	Signif
Box-Jenkins - Estimation by LS Gauss-Newton
Convergence in 2 Iterations. Final criterion was 0.0000000 ≤ 0.0000100
Dependent Variable TRANSFRM

Usable Obs.	1614^b	DF^c	1613^d
Centered R**2	0.998489	R Bar **2	0.998489
Uncentered R**2	0.999998	T x R**2	1613.997

Mean of Dependent Variable 4.5692995052
Std Error of Dependent Variable 0.1598363967
Standard Error of Estimate 0.0062133878
Sum of Squared Residuals 0.0622717807
Log Likelihood 5911.14644
Durbin-Watson Statistic 1.977284
Q(36-1) 23.755838
Significance Level of Q 0.92516272

Variable	Coeff	Std Error	T-Stat	Signif
1. AR{19}	0.0636	0.0249	2.5514	0.0108

^{a}Source: BJetimate_jpyModelBoutput.

^{b}Usable Obs. ("Usable Observations") here is set equal to the number of <u>residuals</u>, T'^r, which equals $T' - \max\{p, sP\} = T - d - sD - \max\{p, sP\} = 1634 - 1\underline{-19}$ (see the footnote immediately below Eq. (3)). See Table 3, too.

^{c}Degrees of Freedom.

^{d}This is equal to Usable Obs. (i.e., T'^r)—the number of parameters excluded (except for the constant) for a block F test, which is ϕ_{19}=1614-1. (For a block F test see Table 13.)

Table 5   Estimated MA(19) Model for First-differenced, Logged Daily JPY/USD: $T = 1634^a$

Box-Jenkins - Estimation by LS Gauss-Newton
Convergence in 5 Iterations. Final criterion was 0.0000062 ≤ 0.0000100
Dependent Variable TRANSFRM

Usable Obs.	1633^b	DF	1632^c
Centered R**2	0.998472	R Bar **2	0.998472
Uncentered R**2	0.999998	T x R**2	1632.997

Mean of Dependent Variable 4.5682758614
Std Error of Dependent Variable 0.1591884754
Standard Error of Estimate 0.0062224164
Sum of Squared Residuals 0.0631885371
Log Likelihood 5978.35535
Durbin-Watson Statistic 1.974103
Q(36-1) 24.091823
Significance Level of Q 0.91755294

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{19}	0.0596	0.0250	2.3886	0.0170

^{a}Source: BJetimate_jpyModelB-MAoutput.

^{b}Usable Obs. ("Usable Observations") here is set equal to $T' - \max\{p, sP\} = T - d - sD - \underline{\max\{p, sP\}} = 1634 - 1\underline{-0}$. See Table 3.

^{c}This is equal to Usable Obs.-the number of parameters excluded (except for the constant), which is θ_{19}=1633-1.

3.2.2 First-differenced, logged daily EUR/USD

First, a white noise model with a constant is estimated to find the constant statistically insignificant at any conventional levels. The constant is thus exclude: See Table 6 and Fig. 14 in Appendix A.

Table 6 Estimated White Noise Model for First-differenced, Logged Daily EUR/USD: $T = 1634^a$

Box-Jenkins - Estimation by LS Gauss-Newton			
Dependent Variable TRANSFRM			
Usable Obs.	1633^b	DF	1633
Centered R**2	0.995759	R Bar **2	0.995761
Uncentered R**2	0.999410	T x R**2	1632.037
Mean of Dependent Variable -0.229888112			
Std Error of Dependent Variable 0.092419859			
Standard Error of Estimate 0.006017117			
Sum of Squared Residuals 0.0591239057			
Log Likelihood 6032.64245			
Durbin-Watson Statistic 2.007230			
Q(36-0) 37.053034			
Significance Level of Q 0.42019584			
NO ESTIMATED COEFFICIENTS			

^{a}Source: BJetimate_eurModelBoutput.
^{b}This equals $T'^r = T' - \max\{p, sP\} = T - d{-}0 = 1634 - 1$. See Table 3.

4 Identification and Estimation: CNY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016)

See Fig. 7 for the first-differenced, logged daily exchange rates, Tuesday, June 22, 2010 - Friday, December 30, 2016.[15] Notice in the figure that, even with the two spikes on Tuesday, August 11 and Wednesday, August 12, 2015, DLOGCNYUSD is far smaller throughout the period than the two others.

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[15]An additional note to the figure: See Fig. 22 in Appendix A for the histograms and scatter diagrams of first differences of the logged daily exchange rates.

Note that the two spikes above will be later referred to at the beginning of Section 5 building a VAR model.

4.1 Not relying on an intervention model: Ignoring permanent level shifts on August 11 and 12, 2015: Identification and estimation for Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016)

Based on Fig. 15 in Appendix A, the first-differenced, logged daily CNY/USD is identified as an MA model with θs at lags 1, 5 and 19, whose estimated results (without a constant) are shown in Table 7 and Fig. 16 in Appendix A.

Notice in Fig. 16 that the residuals are heavily skewed to the right, possibly due to the two permanent level shifts that occurred on August 11 and 12, 2015. See Figs. 18 on (in Appendix A) that draw much less skewed residuals for the intervention models appropriately taking into account the permanent level shifts.

Table 7 Estimated MA Model for First-differenced, Logged Daily CNY/USD: $T = 1634^a$

Box-Jenkins - Estimation by LS Gauss-Newton
Convergence in 6 Iterations. Final criterion was $0.0000067 \leq 0.0000100$
Dependent Variable TRANSFRM

Usable Obs.	1633[b]	DF	1630[c]
Centered R**2	0.998176	R Bar **2	0.998173
Uncentered R**2	0.999999	T x R**2	1632.999

Mean of Dependent Variable 1.8504155345
Std Error of Dependent Variable 0.0329887041
Standard Error of Estimate 0.0014098951
Sum of Squared Residuals 0.0032401209
Log Likelihood 8403.77861
Durbin-Watson Statistic 2.000702
Q(36-3) 34.251367
Significance Level of Q 0.40747296

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	0.0572	0.0247	2.3162	0.0207
2. MA{5}	0.0644	0.0247	2.6087	0.0092
3. MA{19}	0.0430	0.0249	1.7270	0.0844

[a]Source: BJetimate_cnyModelB-MAoutput.

[b]Usable Obs. here is set equal to $T' - \max\{p, sP\} = T - d - sD - \max\{p, sP\} = 1634 - 1 - 0$.

[c]This is equal to Usable Obs.-the number of parameters excluded (except for the constant), which are three MA parameters=1633-3.

4.2 Identification for the Intervention Model: Period V (Monday, June 21, 2010 - Monday, August 10, 2015) (excluding August 11, 2015 on, assuming the model identification here applies to the intervention model for the whole/full period as well)

For how to identify the time series model when the interventions occur, see Doan (2007b, p.337). (For an intervention modeling, including estimation, see Doan 2007b, pp.336-339.)

Based on Fig. 17 in Appendix A, the first-differenced, logged daily CNY/USD is identified as an intervention model with either AR parameters (ϕs at lags 1 and 5) or MA parameters (θs at lags 1 and 5), the latter of which will be estimated (without a constant) as shown in Table 8 and Fig. 18, later in Subsection 4.3.2.

4.3 Estimation of the Intervention Model: CNY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016) (*Including* Tuesday, August 11, 2015 On)

For how to estimate an intervention model, see Kojima (2005, pp.55-71) for the JPYUSD exchange rate behavior, Doan (2007b, pp.336-339) for the the U.S. stock price behavior and Doan (2007a, p.22) for an illustrative intervention in the form of permament level shift.

4.3.1 Intervention model and the iterative procedure of detecting AO and PS

The first half ("...") of the present subsection is quoted from Kojima (2005, pp.56-58) by refering to the original section numbers and table numbers but having the equation numbers and footnote numbers changed:

"Let Z_t denote a time series *not contaminated* (i.e., an *intervention-free* time series) and described by the SARIMA($p, d, q; P, D, s, Q$) model (5), which is here rewritten as:

$$W_{zt} = (1 - B)^d (1 - B^s)^D Z_t$$

$$E[W_{zt}] = \mu$$
$$\tilde{W}_{zt} = W_{zt} - \mu$$
$$\phi(B)\,\Phi(B^s)(1-B)^d(1-B^s)^D Z_t = \theta(B)\,\Theta(B^s)a_t. \tag{9}$$

The model is assumed to satisfy both stationarity and invertibility conditions. The corresponding SARMA$(p, q; P, Q)$ model is (8) with $\tilde{W}_t$ replaced by $\tilde{W}_{zt}$.

Let now X_t^ℓ denote a *contaminated* (logged) observed time series (X_t^ℓ is that given by Eq. (4) in section 3.1). The contaminated series is related with the intervention-free data Z_t as in the following intervention model (Box, Jenkins and Reinsel 1994, ch. 12; RATS UG, pp.277-280[*=Doan 2004b, pp.336-339; Doan 2004a, p.22]* [16]):

$$X_t^\ell = \sum_{k=1}^{m} \omega_{d_k}\left\{\nu_k(B)\xi_t^{(d_k)}\right\} + Z_t, \tag{10}$$

which, put in the differenced form, will be written as

$$\tilde{W}_t = \sum_{k=1}^{m} \omega_{d_k'}\left\{\nu_k(B)\xi_t'^{(d_k')}\right\} + \tilde{W}_{zt} \tag{11}$$

where W_t is a diffrenced series of X_t^ℓ as computed by Eq. (3), m=number of intervention events observed in X_t^ℓ, d_k = point in time when kth intervention event is detected (this notation applies to X_t^ℓ), ω_{d_k} = size of the initial impact of kth intervention event (this applies to $\omega_{d_k'}$ as well), $\xi_t^{(d_k)} = 1$ (for $t = d_k$); $= 0$ (for $t \neq d_k$), and $d_k' = d_k - d - sD$ (this notation applies to differenced series W_t), and $\xi_t'^{(d_k')} = (1-B)^d(1-B^s)^D\xi_t^{(d_k)}$.

In Eqs. (10) and (11), ω are computed by the formulas as shown in Kojima (1994, pp.119-120). $\nu_k(B)$ exhibits different structure depending on type of the kth intervention event (see Kojima 1994, pp.91-94):

AO[17]   If kth intervention event is AO,

$$\nu_k(B) = 1; \tag{12}$$

---

[16]The italic references inside the brackets are newly added for the present paper.

[17]*This footnote is newly added for the present paper: See Fig. 3 for AO that occurred in CNYUSD on Monday, December 19, 1994.*

substituting this into (10) with $m = 1$ leads to

$$X_t^\ell = Z_t, \qquad t \neq \mathrm{d}_k$$
$$X_{\mathrm{d}_k}^\ell = \omega_{A,\mathrm{d}_k} + Z_{\mathrm{d}_{k'}}, \qquad t = \mathrm{d}_k.$$

PS   If kth intervention event is PS (assuming $|B| < 1$),

$$\nu_k(B) = \frac{1}{1-B} = \sum_{i=0}^{\infty} B^i; \qquad (13)$$

substituting this into (10), with $m = 1$, leads to

$$X_t^\ell = Z_t, \qquad t < \mathrm{d}_k$$
$$X_{\mathrm{d}_k+i}^\ell = \omega_{P,\mathrm{d}_k}\psi_i + Z_{\mathrm{d}_k+i}, \qquad i = 0, 1, ...$$

where the ψ weights, equal to 1 here for all i, are those in the random-shock form of the model (9), as given by

$$\tilde{W}_{zt} = \psi(B)a_t \qquad (14)$$

with $\psi(B) = \sum_{i=0}^{\infty} \psi_i B^i$ and $\psi_0 = 1$.[18]
   The number m of intervention events and their observed points in time d_k (or d_k') are unknown in the models (10) or (11). Details are given in Kojima (1994, pp.117-121) on the procedure that will iteratively detect AO and PS and determine m and d_k in outer and inner loops. Just two remarks are in order:
   (i) The series adjusted for the presence of AO and PS (AO-PS adjusted series) is computed by

$$X_t^{\ell*} = X_t^\ell - \hat{\omega}_{\mathrm{d}_k}\left\{\nu_k(B)\xi_t^{(\mathrm{d}_k)}\right\}, \ \mathrm{d}_k = t_{\max}+d+sD \ (t = 1, ..., T);\ ^{19} \ (15)$$

it will be an estimate of unobservable, theoretical Z_t, computed in the final outer loop of the iterative detection procedure. Substituting the random-shock form of (9) with $\mu = 0$ into Eq. (10) yields the general

---

[18]The ψ weights of a general ARIMA model can be recursively determined (*without* relying on any out-of-sample data): see Box, Jenkins and Reinsel (1994, pp.100-102, 139-141). The random-shock form will be again a critical element in the forecasting stage: see Remark 1 in section 6.1.
[19]See Kojima (1994, pp.115-119) for $t_{\max}$ and other related details.

form of an intervention model (to be applied subsequently in section 5.4):

$$X_t^\ell = \sum_{k=1}^m \omega_{d_k} \left\{ \nu_k(B) \xi_t^{(d_k)} \right\} + \frac{\theta(B) \Theta(B^s) / \phi(B) \Phi(B^s)}{(1-B)^d (1-B^s)^D} a_t. \qquad (16)$$

After all the iterations are completed, the intervention model of this form will be estimated simultaneously with regard to all of ω_{d_k}, $k = 1, 2, ..., m$, and parameters ϕ, θ, Φ, Θ. The initial estimates to used then are those estimates obtained in the iterative procedure, $\hat{\omega}_{d_k}$ and parameters computed in the final outer loop. Such simultaneous estimation will be illustrated for the yen-dollar exchange rate in section 5.4.

(ii) Has the intervention model (16) been improved as compared to the model altogether ignoring the intervention events? This problem will be investigated in the forecast performance context in section 6."

See Kojima (2005, p.69) for the actual application of Eq. (16) to the monthly JPY/USD behavior during the sample period of 1987:1 to 2003:12.

For CNY/USD $m = 2$ and the dates d_k, $k = 1, 2$, correspond, respectively, to August 11, 2015 (1287th. observation X_{1287}^ℓ), August 12, 2015 (1288th. observation X_{1288}^ℓ), with ω_{d_k}, $k = 1, 2$ being a magnitude of the respective initial impact of each permanent level shift. By (13), $\nu_k(B) = \frac{1}{1-B}, k = 1, 2$. Rewriting Eq. (16) for the final (revised) model that will be obtained after diagnostic checking and revising the model a couple times in Subsection 4.3.2, thus, one will specify the resultant intervention model as:[20]

$$X_t^\ell = \sum_{k=1,2} \omega_{d_k} \left\{ \frac{1}{1-B} \xi_t^{(d_k)} \right\} + \frac{(1 - \theta_5 B^5)/(1 - \phi_{19} B^{19})}{1-B} a_t \qquad (17)$$

where:

$$k = 1, 2 : \omega_{d_k} \left\{ \frac{1}{1-B} \xi_t^{(d_k)} \right\} = \omega_{d_k} \left\{ \sum_{i=0}^\infty B^i \xi_t^{(d_k)} \right\}$$

$$= \omega_{d_k} \left\{ \xi_t^{(d_k)} + \xi_{t-1}^{(d_k)} + ... \right\}$$

---

[20]See, for example, Eqs. (21) through (23) in Kojima (2005, p.69), which contain AOs (additive outliers) as well.

$$= \begin{cases} 0, & t = \mathrm{d}_k + i, \ i = -1, -2, ... \\ \omega_{\mathrm{d}_k}, & t = \mathrm{d}_k + i, \ i = 0, 1, 2, ... \end{cases} \quad (18)$$

With $W_t^\ell = (1 - B)X_t^\ell$ being interpreted as a rate of change from previous day, Eqs. (17) and (18) combined together are rewritten, in the form of Eq. (11), as:

$$W_t^\ell = \sum_{k=1,2} \omega_{\mathrm{d}_k'} \left\{ \xi_t'^{(\mathrm{d}_k')} + \xi_{t-1}'^{(\mathrm{d}_k')} + ... \right\} + \phi_{19} W_{t-19}^\ell + a_t - \theta_{t-5}a_{t-5} \quad (19)$$

The notation for Eq. (19) is related to that for Eq. (17) as quoted earlier: With $T = 1634$ denoting the sample size (the end of the sample period for X_t^ℓ), the effective sample size (the effective end of the first-differenced series W_t^ℓ) is $T' = T - 1 = 1633$;[21] d_k = point in time when kth intervention event is detected (this notation applies to X_t^ℓ), ω_{d_k} = size of the initial impact of kth intervention event (this applies to $\omega_{\mathrm{d}_k'}$ as well), $\xi_t^{(\mathrm{d}_k)} = 1$ (for $t = \mathrm{d}_k$) or $= 0$ (for $t \neq \mathrm{d}_k$), and $\mathrm{d}_k' = \mathrm{d}_k - 1$ (this notation applies to the first-differenced series W_t^ℓ), and $\xi_t'^{(\mathrm{d}_k)} = (1 - B)\xi_t^{(\mathrm{d}_k)}$;

$$\mathrm{d}_1 = 1287 \text{ (August 11, 2015: } \mathrm{d}_1' = \mathrm{d}_1 - 1 = 1286\text{)},$$
$$\mathrm{d}_2 = 1288 \text{(August 12, 2015: } \mathrm{d}_2' = \mathrm{d}_2 - 1 = 1287\text{)};$$

by Eq. (18),

$$\text{the summation for } \omega_{\mathrm{d}_k'} = \begin{cases} \omega_{1286}, \text{ only for } t = 1286 \text{(August 11, 2015)}, 1287, \cdots \\ \omega_{1287}, \text{ only for } t = 1287 \text{(August 12, 2015)}, 1288, \cdots \end{cases} (20)$$

$$\text{the summation for } \omega_{\mathrm{d}_k} = \begin{cases} \omega_{1287}, \text{ only for } t = 1287 \text{(August 11, 2015)}, 1288, \cdots \\ \omega_{1288}, \text{ only for } t = 1288 \text{(August 12, 2015)}, 1289, \cdots \end{cases} (21)$$

For *not* including the constant c either in (17) or its first-differenced version (19) for W_t^ℓ, see the footnote attached to Eq. (7) containing the constant.

In the following subsection, after diagnostic checking and revising the model a couple times, we will finally estimate Eq. (19) or Eq. (17) for $\omega_{\mathrm{d}_k'}$ or ω_{d_k} $(k = 1, 2)$, for $\mathrm{d}_k' = 1286$(August 11, 2015), 1287(August 12, 2015) in Eq. (20) or for $\mathrm{d}_k = 1287$(August 11, 2015), 1288(August 12, 2015) in Eq. (21) as well as the AR and MA parameters.

---

[21] See the footnote on T, T', and T'^r, respectively, for the raw data, the differenced data and the <u>residuals</u> series for a univariate SARIMA$(p, d, q; P, D, s, Q)$ model in Subsection 3. For T'^r for the current model, see also Table 10 in the present subsection.

The two dummies associated with the two parameters $(\omega_{d'_k}, k = 1, 2)$ in Eq. (20) are displayed in Table 11 along with the parameter estimates in Table 10; see the note in Table 11.

4.3.2 Estimation of the Intervention Model (16) or its first-differenced version for W_t^ℓ

For *not* including the constant c in (16) or its differenced version for W_t^ℓ, see the footnote attached to Eq. (7) containing the constant.

Diagnostic checking and revising   The intervention model with MA parameters (θs at lags 1 and 5) is estimated (without a constant) as shown in Table 8 and Fig. 18 in Appendix A: θ_5 turns out statisitcally insignificant at any conventional levels and hence is excluded next in Table 9 and Fig. 19 in Appendix A.

Table 8 [First Version] Estimated Intervention Model for First-differenced, Logged Daily CNY/USD: Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016); $T = 1634^a$

```
Box-Jenkins - Estimation by LS Gauss-Newton
Convergence in 6 Iterations. Final criterion was 0.0000056 ≤ 0.0000100
Dependent Variable TRANSFRM
Usable Obs.            1633ᵇ         DF            1629ᶜ
Centered R**2         0.998416      R Bar **2     0.998413
Uncentered R**2       0.999999      T x R**2      1632.999
Mean of Dependent Variable 1.8504155345
Std Error of Dependent Variable 0.0329887041
Standard Error of Estimate 0.0013141824
Sum of Squared Residuals 0.0028134058
Log Likelihood 8519.08055
Durbin-Watson Statistic 1.999421
Q(36-2) 47.505426
Significance Level of Q 0.06189589
```

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	-0.003	0.0248	-0.1062	0.9154
2. MA{5}	0.0763	0.0247	3.0873	0.0021
3. N_PSAUG112015{0}^{d}	0.0186	0.0013	14.1560	0.0000
4. N_PSAUG122015{0}	0.0099	0.0013	7.5539	0.0000

^{a}Source: BJetimate_cnyModelB-MAoutput_forIdentify1-1286.

^{b}This equals $T'^r = T' - \max\{p, sP\} = T - d\underline{-0} = 1634 - 1$. See Table 3 and the footnote in Table 4.

^{c}This is equal to Usable Obs.-the number of parameters excluded (except for the constant), which are four (i.e., $\theta_1, \theta_5, \omega_{d_k}$, $k = 1, 2$)=1633-4.

^{d}N_PSAUG112015{0}   denotes $\omega_{\text{August 11, 2015}}$, N_PSAUG122015{0} $\omega_{\text{August 12, 2015}}$, in Eqs. (20) and (21).

Table 9 [Second Version] Estimated Intervention Model for First-d-ifferenced, Logged Daily CNY/USD: Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016); $T = 1634^a$

Box-Jenkins - Estimation by LS Gauss-Newton
Convergence in 6 Iterations. Final criterion was $0.0000016 \leq 0.0000100$
Dependent Variable TRANSFRM

Usable Obs.	1633	DF	1630
Centered R**2	0.998416	R Bar **2	0.998414
Uncentered R**2	0.999999	T x R**2	1632.999

Mean of Dependent Variable 1.8504155345
Std Error of Dependent Variable 0.0329887041
Standard Error of Estimate 0.0013137836
Sum of Squared Residuals 0.0028134244
Log Likelihood 8519.07516
Durbin-Watson Statistic 2.004549
Q(36-1) 47.413490
Significance Level of Q 0.07850778

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{5}	0.0765	0.0247	3.0942	0.0020
2. N_PSAUG112015{0}	0.0186	0.0013	14.1616	0.0000
3. N_PSAUG122015{0}	0.0099	0.0013	7.5525	0.0000

^{a}Source: BJetimate_cnyModelB-MA_IntrvModel2output.

Further diagnostic checking and revising: The final model   Fig. 19 (the bottom-left SCCF in particular) in Appendix A shows that AR parameter ϕ_{19} is to be added: The further revised intervention model is thus estimated and its results are shown in Table 10 and Fig. 20 in Appendix A.

Over 1.8% devaluation from the previous day, on August 11, 2015 [the estimated $\omega_{d'_1}$ or ω_{d_1}, a magnitude of the initial impact of the first permanent level shift on August 11]; nearly 1% devaluation from the previous day, on August 12, 2015 [the estimated $\omega_{d'_2}$ or ω_{d_2}, a magnitude of the initial impact of the second permanent level shift on August 12]; thus, nearly 2% devaluation through the two day period on August 11 and August 12.[22]   These percentage figures both approximate the corresponding daily (exact) rates of change in the raw CNY/USD (see Table 12, which also refers to first-differenced, logged CNY/USD on the

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[22]For this interpretation of the estimated coefficients for the permanent level shifts using the first-differenced logged series, see Doan (2004b, pp.338-339).

two dates).

Table 10 [Third, Final Version] Estimated Intervention Model for First-differenced, Logged Daily CNY/USD: Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016); $T = 1634^a$

Box-Jenkins - Estimation by LS Gauss-Newton				
Convergence in 7 Iterations. Final criterion was $0.0000036 \leq 0.0000100$				
Dependent Variable TRANSFRM				
Usable Obs.	1614^b	DF	1610	
Centered R**2	0.998359	R Bar **2	0.998356	
Uncentered R**2	0.999999	T x R**2	1613.999	
Mean of Dependent Variable 1.8496611173				
Std Error of Dependent Variable 0.0324357792				
Standard Error of Estimate 0.0013151733				
Sum of Squared Residuals 0.0027847860				
Log Likelihood 8418.76768				
Durbin-Watson Statistic 2.013424				
Q(36-2) 45.604476				
Significance Level of Q 0.08820633				
Variable	Coeff	Std Error	T-Stat	Signif
1. AR{19}	0.0404	0.0251	1.6137	0.1068
2. MA{5}	0.0732	0.0249	2.9434	0.0033
3. N_PSAUG112015{0}	0.0185	0.0013	14.0822	0.0000
4. N_PSAUG122015{0}	0.0098	0.0013	7.5040	0.0000
Statistics on Series RESIDS				
Observations 1614^c				
Sample Mean -0.000001 Variance 0.000002				
Standard Error 0.001314 of Sample Mean 0.000033				
t-Statistic (Mean=0) -0.024444 Signif Level 0.980501				
Skewness 0.154087 Signif Level (Sk=0) 0.011574				
Kurtosis (excess) 5.879190 Signif Level (Ku=0) 0.000000				
Jarque-Bera 2330.874394 Signif Level (JB=0) 0.000000				

^{a}Source: BJetimate_cnyModelB-ARMA_IntrvModeloutput.

^{b}This equals $T''^r = T' - \max\{p, sP\} = T - d\underline{-p} = 1634 - 1\underline{-19}$. See Table 3 and the footnote in Table 4. For the model (19), $T' = T - 1$ and $T''^r = T' - \max\{p, sP\}$; thus the differenced data start at $1 + d + sD = 1 + 1 = 2$ and the residuals at $1 + d + sD + \max\{p, sP\} = 1 + 1\underline{+19} = 21$.

^{c}See the footnote immediately above.

Displayed in Table 11 are the two dummies, PSAUG112015 and PSAUG12201 associated, respectively, with N_PSAUG112015{0} and N_PSAUG122015{0}, the two estimates for $\omega_{\text{August 11, 2015}}$ and $\omega_{\text{August 12, 2015}}$.

Table 11 Two Dummies Associated with the Two Parameter Estimates, N_PSAUG112015{0} and N_PSAUG122015{0}, in Table 10

ENTRY(for Eq. (17))	ENTRY$'$(for Eq. (19))	PSAUG112015	PSAUG122015
1		0	0
2	1	0	0
3	2	0	0
⋮	⋮	⋮	⋮
1284	1283	0	0
1285	1284	0	0
1286	1285	0	0
1287(= d_1)	1286(= d'_1)	1	0
1288(= d_2)	1287(= d'_2)	1	1
1289	1288	1	1
1290	1289	1	1
⋮	⋮	⋮	⋮
1633	1632	1	1
1634(= T)	1633(= T')	1	1

Note in Table 10 that for the model (19), $T' = T - 1$ and thus the differenced data start at at $1 + d + sD = 1 + 1 = 2$ in the present table (and thus $d'_k = d_k - 1$).

Table 12 Daily (Exact) Rates of Change in the Raw CNY/USD

Date (in 2015)	Raw CNY/USD	Daily (Exact) Rate of Change	First-differenced, Logged CNY/USD[a]
Aug. 7	6.2101		
Aug. 10	6.2083	-0.00029 or -0.029%	-0.000289892420
Aug. 11	6.3242	0.01867 or 1.867%	0.018496437945
Aug. 12	6.3875	0.01001 or 1.001%	0.009959411129
Aug. 13	6.3982	0.00168 or 0.168%	0.001673745278

[a]For its plot for Tuesday, June 22, 2010 - Friday, December 30, 2016, see Fig. 7. See Subsection 3.1 for the economic interpretation of first-differenced, logged series as a rate of change.

5   VAR Modeling: Period V (Monday, June 21, 2010 - Monday, August 10, 2015)

For the sample period for VAR modeling in the present section, see Table 2 and a note on the shaded period in Figs. 1 through 6; the corresponding three daily exchange rates are drawn in Fig. 8 ($T = 1286$).

Note in particular that the period beyond Monday, August 10, 2015 is not considered in the present study, since both on August 11 and 12, 2015, the central bank in China unanticipatedly and heavily controlled the Yuan by significantly devaluing it;[23] this naturally increased its variability during the period extending beyond August 10 (see Figs. 5 through 7).

5.1 Why VAR modeling

Why VAR modeling for the three daily exchange rates, JPY/USD, EUR/USD and CNY/USD? There are two types of VAR to be studied: The unrestricted VAR and the cointegrated VAR (or VECM).

If, *a priori* (*deductively*), there are equilibrium condition to be satisfied by the three daily exchange rates, JPY/USD, EUR/USD and CNY/USD, then the cointegrated VAR (or VECM) is the one to be used to test for the equilibrium condition (or the cointegration relation).

If there are no such condition or restriction a priori, then the unrestricted VAR may be more appropriate. The present study presents no equilibrium condition a priori and thus will rely on the unrestricted VAR. Harris (1995, p.117) argues that "prior information motivated by economic arguments forms the basis for imposing restrictions(.)" The present paper apparently fails to provide such prior information for the three daily exchange rates.

Yet Figs. 21 and 22 in Appendix A would be useful to *a posteriori* (*inductively*) look at any possibility of correlations among the three daily exchange rates (respectively, logged and first-differenced logged ones) during the sample period V (Monday, June 21, 2010 - Monday, August 10, 2015). Fig. 21 may show that, with a possibility of spurious correlations, the contemporaneous relations in level are positive between JPY/USD and EUR/USD, whereas negative between JPY/USD and CNY/USD and between EUR/USD and CNY/USD. Excluding such spuriosity, Fig. 22 evidences no contemporaneous relations (in rates of change) between any pair of the three Jexchange rates, implying that, a posteriori, no equilibrium condition or cointegration relation appears to be detected.

To confirm this a posteriori finding, It would be useful to conduct a cointegration test: See Subsection 5.2.

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[23] See Section 4 for the (univariate) intervention analysis of the spikes on the two dates.

5.2 Cointegration test: Two-step tesing procedure

To confirm the *a posteriori* (graphical) inference of no equilibrium condition or cointegration relation in the preceding subsection 5.1, the present subsection will formally conduct a cointegration test: Two-step tesing procedure as proposed by Doan (2007b, p.254).

Taking two series, each of which has a unit root, will in general lead to a linear combination of them also having a unit root: This indeed appears to be the case for the three exchange rates under study, based on the preceding subsection 5.1. Yet there will possibly exist a linear combination that is stationary; if so, then the two series are said to be *cointegrated* **and such a specific linear combination may be termed a "restriction" (that is rooted in economic theory) (Engle and Granger 1987; Doan 2007b, pp.252-253). The present subsection will thus formally do two-step cointegration tests for the three exchange rates.**

The number of lags to be considered at both Step 1 (in Subsection 5.2.1) and Step 2 (in Subsection 5.2.2) below is set equal to $12 \times (T/100)^{1/4}$, which is 22 (days) with $T = 1286$.[24] At Step 2, though, the *single-equation* based, two-stage Engle-Granger (1987) test of cointegration will use the lag length of 22 and yet the *(multivariate) VAR* based Johansen (1988, 1991) likelihood ratio test (at the same step) will set the lag length equal to 2 as well as 22.[25]

5.2.1 Step 1: Testing the individual series for unit roots: (Usual) Univariate testing

All the individual series MUST pass (that is, all must be non-stationary) before moving on to Step 2 (see Harris 1995, p.55 and Doan 2007b,

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[24]The formula is applied for testing for unit roots to quarterly data by Schwert (1989, p.151) and Harris (1995, p.36).

[25]Referring to Hamilton (1994), Doan (2007b, pp.253-254) illustrates an empirical testing whose lag length is longer both for the univariate unit root testing at Step 1 and the single-equation based Engle-Granger (1987) test at Step 2 than for the (multivariate) VAR based Johansen (1988, 1991) likelihood ratio test at Step 2.

For the purposes of the VAR modeling in Subsection 5.3.1 the lag length will be tested following Doan (2007b, pp.348-349): The VAR modeling there in Subsection 5.3 will set the lag length equal to 2, not 22. Setting the lag length equal to 2 as well as 22, the Johansen (1988, 1991) likelihood ratio test (at Step 2 in Subsection 5.2.2) will see if the cointegration test results would differ between the two: No differences will be found, as will be documented in the tables there.

p.252).[26]

(Usual) univariate testing for a unit root, with the null of a unit root (i.e., non-stationarity) conducted in Tables 17 (based on Eq. (28)) and 18 in Appendix B show that all the individual series pass the unit root tests (that is, all are found non-stationary).[27]

5.2.2   Step 2: The proposed "equilibrim" condition is tested, using the testing procedure for cointegration with an *unknown* cointegrating vector

Note that since the present paper proposes no such (a priori) equilibrium condition, cointegration with an *unknown* cointegrating vector is studied in the present subsection.[28]

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[26]Hansen and Juselius (1995, p.1) remark that "... we assume that y_t is at most $I(1)$ ... However not all the individual variables included in y_t need be $I(1)$, as is often incorrectly assumed. To find cointegration between nonstationary variables, only two of the variables have to be $I(1)$." See also Kojima (2006a, p.8; 2010, p.72).

[27]Specifically, to be estimated with $p = 22$ for DF and augmented DF tests, regression equations in which $u_t \sim IID(0, \sigma^2)$ are as follows (Harris 1995, pp.28-30, 32-34; Enders 2004, pp.181-182, for example):

$$\Delta y_t = \gamma y_{t-1} + u_t \text{ [a pure random walk model]} \tag{22}$$

$$\Delta y_t = \underline{a_0 +}\gamma y_{t-1} + u_t \text{ [with a drift term added]} \tag{23}$$

$$\Delta y_t = \underline{a_0 +}\gamma y_{t-1}\underline{+a_2 t} + u_t$$
$$\text{[with a drift term and a linear trend term added];} \tag{24}$$

assuming y_t follows a pth order AR process (Harris 1995, p.32)

$$y_t = \psi_1 y_{t-1} + \cdots + \psi_p y_{t-p} + u_t, \tag{25}$$

the augmented version is, with $\psi^* = \Sigma_{i=1}^p \psi_i - 1$,

$$\Delta y_t = \psi^* y_{t-1} + \Sigma_{i=1}^{p-1} \psi_i^* \Delta y_{t-i} + u_t \tag{26}$$

$$\Delta y_t = \underline{a_0 +}\psi^* y_{t-1} + \Sigma_{i=1}^{p-1} \psi_i^* \Delta y_{t-i} + u_t \tag{27}$$

$$\Delta y_t = \underline{a_0 +}\psi^* y_{t-1} + \Sigma_{i=1}^{p-1} \psi_i^* \Delta y_{t-i}\underline{+a_2 t} + u_t. \tag{28}$$

If the null of $\gamma = 0$ or $\psi^* = 0$ is not rejected, then the y_t series contains a unit root.

Notice the number of *lags on the differences on the right-hand side* is $p - 1$. (The lag structure for the differences here will be later contained in the VEC model (30) in Subsection 5.2.2.)

"If the residuals of a unit root process are heterogeneous or weakly dependent, the alternative Phillips-Perron (1988) test can be used (Enders 2004, p.229); see also Harris (1995, pp.33-34) for the Phillips-Perron (1988) test.

[28]The testing procedure for a *known* cointegration vector as well as this "unknown" case is described in Doan (2007b, p. 253). The cointegration vector may be assumed

Tables 19 - 24 in Appendix B (Table 21 including a deterministic trend term and Table 24 excluding a deterministic trend term, in particular, at Stage 2b), generated and complied based on Engle and Granger (1987) proposing a two-stage residual-based ADF test for cointegration with the null of *no* cointegration and the alternative of cointegration (Harris 1995, pp.53-55), all show that the null of a unit root and thus *no* cointegration (that is, *no* a priori restrictions imposed) is *not* rejected for the three daily exchange rates. (See also Doan 2007b, p.254, Example 6.8.)

This finding may not be unexpected, for a priori there would exist equilibrium condition that will make the three daily exchange rates cointegrated. Yet the present subsection is attempting to a posteriori find such equilibrium condition but apparently, as will be seen from Tables 21 and 24 in particular, fails to do so. Recall from the first few paragraphs in Subsection 5.1 that the present paper apparently fails to provide "prior information motivated by economic arguments that forms the basis for imposing restrictions(.)" (Harris 1995, p.117) for the three daily exchange rates.

Subsection 5.3 will, therefore, build *unrestricted* VAR models (that is, VAR models with no a priori restrictions) for the the three daily exchange rates.

Two two-stage Engle-Granger (1987) tests of cointegration that follow are the one including a deterministic trend term and the other excluding a deterministic trend term, though Harris (1995, pp.53-54) suggests no such a trend term is to be included. See Harris (1995, p.57) for why the Engle-Granger (1987) testing procedure is so popular, while inherently having several inference-related problems; for the problems here see also Harris (1995, pp.62-66, 72) asserting the "(r)ather, the multivariate VAR approach developed by Johansen (1988, 1991) is the more obvious place to begin testing for cointegration." Thus, the Johansen likelihood ratio test will follow the Engle-Granger (1987) tests.

Single-equation based, two-stage Engle-Granger (1987) test of cointegration, with a drift term and a linear trend term added: The regression-based test [29]

Table 19 (Stage 1) and Tables 20 and 21 (Stage 2)[30] in Appendix B show, as summarized above, that the null of a unit root and thus no

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known when equilibrium condition is a priori proposed.

[29]See Harris (1995, pp.52-54) and Doan (2007b, pp.253-254).

[30]How the tables are produced is briefly described by Doan (2007b, p.253).

cointegration (that is, no a priori restrictions imposed) is not rejected for the three daily exchange rates. (See also Doan 2007b, p.254, Example 6.8.)

Notice in the tables "Dependent Variable LOGJPYUSD." The results for "Dependent Variable LOGEURUSD" and "Dependent Variable LOGCNYUSD" turn out all similar to those below; this applies to the two-stage Engle-Granger test of cointegration, WITHOUT determ=trend.[31]

Single-equation based, two-stage Engle-Granger (1987) test of cointegration, with a drift term added but without a linear trend term: The regression-based test [32]

Table 22 (Stage 1) and Tables 23 and 24 (Stage 2) in Appendix B show, as summarized earlier in the present subsection, that the null of a unit root and thus no cointegration (that is, no a priori restrictions imposed) is not rejected for the three daily exchange rates.

(Multivariate) VAR based, Johansen (1988, 1991) likelihood ratio test of cointegration, with a drift term, a cidrift term or no deterministic term: The likelihood ratio approach (the bassis for the CATS software) [33]

The following paragraphs within "..." of the present subsection is quoted from Kojima (2006a, pp.8-10; 2010, pp.72-73) by refering to the original section numbers and table numbers in the former paper but having the equation numbers, the footnote numbers and the reference years for Doan changed (with Doan 2004a replacing the original Doan *RM*, for example):

"Multivariate cointegration tests of PPP are conducted using the Johansen method, to examine the long-run structure (the number of unit

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[31] See the text files CointegrTests_output2 and CointegrTests_output3, saved in the folder 'Business Forecasting&PanelDtAnlys (Incld. Anderson): Yr2016o_new research: Published in 2018: MacRATS: charts (pdf) and numberical output (txt) saved).' The results (except the signs for 'Cointegrating Vector for Largest Eigenvalue") for Johansen likelihood ratio tests, WITH determ=trend and WITHOUT determ=trend, remain unchanged irrespective of the dependent variables chosen for the two-stage Engle-Granger tests here. The two text files above are available from the author upon request.

[32] See Harris (1995, pp.52-54) and Doan (2007b, pp.253-254).

[33] See Estima (2012, pp.18-29) which describes how to use CATS 2.0 to reproduce the results from Juselius (2006).

roots)[34] in the vector $\boldsymbol{y}_t = (s_t, p_t^*, p_t)'$, each element of which is a potentially endogenous variable[35] and assumed to be integrated of order 1, $I(1)$.[36] . To conduct the tests, we consider the VAR model including a constant and augmented with centered seasonal dummies:[37]

$$\boldsymbol{y}_t = \sum_{l=1}^{L} \boldsymbol{\Phi}_l \boldsymbol{y}_{t-l} + \boldsymbol{\mu} + \boldsymbol{\Psi} \boldsymbol{D}_t + \boldsymbol{u}_t. \tag{29}$$

The underlying VAR model is reformulated in the error-correction form as the VEC model:[38]

$$\Delta \boldsymbol{y}_t = \sum_{l=1}^{L-1} \boldsymbol{\Phi}_l^{\Delta} \Delta \boldsymbol{y}_{t-l} + \boldsymbol{\Pi} \boldsymbol{y}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\Psi} \boldsymbol{D}_t + \boldsymbol{u}_t \tag{30}$$

where: Δ is the first-difference operator; the *short-run* matrices $\boldsymbol{\Phi}_l^{\Delta}$ represent the short-run dynamics/adjustment to past change in $\boldsymbol{y}_t$, $\Delta \boldsymbol{y}_{t-l}$;[39] and the *long-run* marix $\boldsymbol{\Pi}$ represents long-run adjustment. The initial assumptions include, in particular, the white noise $\boldsymbol{u}_t \sim IN(\boldsymbol{0}, \boldsymbol{\Sigma})$ or $\boldsymbol{u}_1, ..., \boldsymbol{u}_T$ are $niid(\boldsymbol{0}, \boldsymbol{\Sigma})$; the dependence is allowed among the white-noise disturbance terms $u_{1t_1}, u_{2t_2}, u_{3t_3}$ for any $t_i, i = 1, 2, 3$. For monthly

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[34]For the long-run structure and/or the number of unit roots, see ⊂ 3 ⊃ in section 3.1.2, the paragraph "Restricting, jointly, $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$" in section 3.2.2, and the paragraph "Long-run structure and real exchange rate" in section 3.2.3.

[35]It could turn out wealky exogenous, as will be evidenced in section 3.2.2. The rationale behind choosing the ordering (s_t, p_t^*, p_t) instead of, for example, its reverse (p_t, p_t^*, s_t) is given in section 5.2.

[36]Hansen and Juselius (1995, p.1) remark that "... we assume that $\boldsymbol{y}_t$ is at most $I(1)$ ... However not all the individual variables included in $\boldsymbol{y}_t$ need be $I(1)$, as is often incorrectly assumed. To find cointegration between nonstationary variables, only two of the variables have to be $I(1)$."

[37]For *centered* seasonal dummies, see Hansen and Juselius (p.8) and Doan (2004a, p.84, pp.367-368); Harris (1995, p.81) remarks that "Seasonal dummies are centered to ensure that they sum to zero over time, and thus they do not affect the underlying asymptotic distributions upon which tests (including tests for cointegration rank) depend."

[38] *This footnote is newly added for the present paper: The term $\boldsymbol{\Pi} \boldsymbol{y}_{t-1}$ may be written more generally as $\boldsymbol{\Pi} \boldsymbol{y}_{t-a}$ where a "is an integer between 1 and L (or p) defining th lag plaxenent of the ECM (error correction model) term. Note that the value of the likelihood function does not change even if we change the value of a."* (Juselius 2006, pp.61-66).

Also, the lag structure for the differences here in Eq. (30) is similar to that in Eq. (28) in Subsection 5.2.1.

[39]The terms "short-run matrices" and "short-run dynamics" are those used by Hansen and Juselius (ps.29, 71).

data, L may be set at 12; it will be far smaller for our set of the data, however, as shown later.

Short-run effects/dynamics/matrices $\mathbf{\Phi}_l^{\triangle}$, the short-run dynamics/adjustment to past changes in $\boldsymbol{y}_t$, and their estimates are crucial in our analysis of short-run PPP, for C-R-X has shown, using the pure inflation rate they extracted from stock returns, that the short-run PPP is strongly supported.

Note that the analysis of the short-run structure (consisting of short-run effects $\mathbf{\Phi}_l^{\triangle}, l = 1, ..., L - 1$) here will be made *after* the modeling of the long-run structure is completed: the estimated cointegration vectors in the long-run structure will be considered as given or known, in the subsequent short-run analysis (in section 4).

Long-run adjustment   If the long-run marix $\mathbf{\Pi}$ is either zero or non-zero and full-rank, it is of no use to write the VAR in form (30) rather than (29), to begin with; if it is non-zero but less than full-rank, then it is usefully written as[40]

$$\mathbf{\Pi} = \boldsymbol{\alpha\beta}' \qquad (31)$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $3{\times}r$ matrices, with r being the rank of $\mathbf{\Pi}$.[41] Following Engle and Granger's (1987) definition of equilibrium error,[42] $\boldsymbol{\beta}'\boldsymbol{y}_{t-1} \neq 0$ in eq. (30) is interpreted as an equilibrium error, with $\boldsymbol{\beta}$ being a matrix representing long-run coefficients such that the term $\boldsymbol{\beta}'\boldsymbol{y}_{t-1}$, the deviation from long-run equilibrium embedded in eq. (30), represents up to $(n-1)$ cointegration relationships in the multivariate model which ensure that the $\boldsymbol{y}_t$ converge to their long-run steady-state solutions. The rank r indicates the number of cointegration relations $\boldsymbol{\beta}'\boldsymbol{y}_{t-1}$. Assuming $\boldsymbol{y}_t$ is a vector of nonstationary $I(1)$ variables, then all the terms in (30) which involve $\Delta\boldsymbol{y}_t$ are $I(0)$, while $\mathbf{\Pi}\boldsymbol{y}_{t-1}$ must also be stationary for

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[40]See Doan (2004b, p.360). In this case, eq. (30) *without* the term $\mathbf{\Pi}\boldsymbol{y}_{t-1}$ would be a misspecified model.

[41]$\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are matrices of full rank (see Hansen and Juselius, p.2). The decomposition in eq. (31) is not unique; where r is one, it is unique up to a scale factor in the two parts (see Doan 2004b, p.360).

[42]The very beginning of Engle-Granger's formal analysis is to consider a set of n economic variables in long-run equilibrium when $\sum_{i=1}^{n} \beta_i y_{it} = 0$; the equilibrium error is a disequilibrium defined as a deviation from long-run equilibrium and given by $e_t \equiv \sum_{i=1}^{n} \beta_i y_{it}$; in the long run, $e_t = 0$.

$u_t \sim I(0)$ to be white noise (Harris 1995, p.79).[43]

α is a matrix representing a measure of the average speed of convergence towards the long-run equilibrium (i.e., the speed of adjustment to disequilibrium).[44] The elements of α will be shown in section 3.2.3 to indicate how rapidly a current deviation from PPP is offset in the future.

Π, the long-run adjustment, has been the major topic of interest in the cointegration and error-correction model analysis of PPP. In a way, this is due to the lack of short-run support for PPP in the past PPP literature. Now that C-R-X have found strong support for PPP in the short-run and made available more appropriate inflation rate data for the first time, it is an interesting and meaningful work, using their extracted price data, to statistically examine the short-run dynamics based on the VAR model and its error-correction representation."

Using the terminology used above in "...," Harris (1995, p.79, p.88) may be rephrased, with n denoting the number of potentially endogenous variables, as follows: "(I)f the *long-run* marix Π has full rank (i.e., there are $r = 3(= n)$ linearly independent columns), then the variables in y_t are $I(0)$, while if the rank of Π is zero then there are no cointegration relationships. Neither of these two cases is particularly interesting. More usually, y_t has reduced rank; that is, there are $r \leq 2(= n - 1)$ cointegration vectors present. Later on we shall cconsider actual tests for the (reduced) rank of Π, ..." The tests constitute the Johansen reduced rank regression approach, which are now conducted for the three exchange rates.

Relying on the trace statistics (rank test statistics) "Trace," the bottom panels of Tables 25 and 26 (with the number of lags being set equal to 2, as will be shown in Subsection 5.3 to be appropriate for VAR modeling) and those of Tables 27 through 29 all (with the number of lags being set equal to 22), in Appendix B, show that, based on P-Value:

(i) Every *null* hypothesis of p-r unit roots (p-r= 3, 2, 1)[45] is easily accepted or not rejected (though at the 5% and 1% levels of siginificance for the p-r= 3 unit roots in Table 26 in Appendix B);[46]

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[43] That the deviation from long-run equilibrium is stationary means that the deviation is temporary in nature. The stationarity requirement imposed on Πy_{t-1} is investigated by Kojima (2006b).

[44] See Hansen and Juselius (pp.2-3) and Harris (1995, pp.77-78).

[45] The symbol p in the tables is not p, the order of VAR, bur rather n, in the text, while r is the same as r in the text.

[46] See Estima (2012, p.29).

(ii) in other words,[47] accepted are the *null* of no more than (i.e., fewer than or equal to) zero cointegrating vector (r≤ 0 or r= 0, that is, no cointegrating vectors) against the alternative of one or more cointegrating vectors (r> 0) (though at the 5% and 1% levels of siginificancein Table 26 in Appendix B),

the nul of no more than (i.e., fewer than or equal to) one cointegrating vector (r≤ 1) against the alternative of two or more cointegrating vectors (r> 1), and

the nul of no more than (i.e., fewer than or equal to) two cointegrating vector (r≤ 2) against the alternative of three cointegrating vectors (r> 2).

Whether the number of lags is set equal to 22 or 2, **it thus follows that there are detected as many as three unit roots, in other words, no cointegrating vectors: No cointegration relationships among the three daily exchange rates are detected. This is consistent with the earlier result of single-equation based, two-stage Engle-Granger (1987) tests of cointegration, with or without a linear trend term added**, as summarized at the beginning of the present subsection. We will, therefore, turn to the unrestricted VAR models (that is, VAR models with no a priori restrictions) in the following subsection, where the appropriate number of lags will be shown to be equal to 2.

5.3   Unrestricted VAR modeling

Based on the cointegration tests in the preceding Subsection 5.2, the present subsection assumes no a priori equilibrium condition or cointegration relation imposed on the the three daily exchange rates and will build *unrestricted* VAR models, to study whether or not each lagged exchange rate is still to be included in the (entire) VAR model even with no a priori equilibrium condition or cointegration relation, thereby exploring for the possibility of the three exchange rates behaving jointly during the period V (Monday, June 21, 2010 - Monday, August 10, 2015), when the Chinese Yuan was continuously less managed/controlled by the central bank in China under (managed) flexible exchange rate system (see Section 1 referring to Tables 1 and 2).

The present subsection consists of further subsections on the lag length, preliminary transformations, levels or differences, trend or no trend, and

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[47]See Enders (2004, pp.364-365).

F and chi-squared tests.

5.3.1 Setting the lag length

The test results in Tables 30 and 31 in Appendix B, combined together, show that the appropriate lag length for the daily exchange-rate VAR models is as short as 2 (days).[48]

The lag length detected here for the VAR modeling sharply differs from the one (that is, 22 days) detected for testing for unit roots by Harris' (1995, p.36) approach relying on Schwert's (1989, p.151) formula: See the footnote at the beginning of Subsection 5.2. Notice in Table 31 in particular that the null of (shorter) lags= 2 is easily accepted against the alternative of (longer) lags= 20: The VAR modeling here thus sets the lag length equal to 2 (not 22).

5.3.2 Preliminary transformations; levels or differences; trend or no trend

The unrestricted VAR model to be studied is thus Eq. (29) with $n = 3$ and $L = 2$ but without the term $\boldsymbol{\Psi D}_t$:

$$\boldsymbol{y}_t = \sum_{l=1}^{2} \boldsymbol{\Phi}_l \boldsymbol{y}_{t-l} + \boldsymbol{\mu} + \boldsymbol{u}_t, \tag{32}$$

which is written based on Doan (2007b, pp.343-344):

"You will usuall leave in levels non-trending series, such as interest ... rates ... in a VAR including prices and exchange rates (which should be in logs) ...."

"In Box-Jenkins modeling for single series, appropriate differencing is important for several reasons. ... Neither of these applies to VAR's. ... In a VAR, differencing throws information away (for instance, a simple

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[48]See Doan (2007b, p.344): "Where we are including an identical number of lags on all variables in all equations, the number of pamperers goes us very quicky–we add N^2 parameters with each new lag. Beyond the first lag or two, most of these new additions are likelyo be unimportant, causing the information criteria to reject the longer lags in favor of shorter ones. ... It is also possible to use a systematic procedure to choose a different number of lags for each variable in each equation. ... The use of priors is an alternative to relying on short lags or data-driven selection methods. If data are adequate, it is recommended that you include at least a year's worth of lags."

VAR on differences canot capture a cointegrating relationship), while it produces almost no gain."

"In most economic time series, the best representation of a trend is a random walk with drift .... Because of this, we would recommend against including a deterministic trend term in your VAR."

5.3.3 Tests on three differing nulls of lagged regressor(s) being *excluded/omitted*

Three Tests [F], [C1] and [C2] are conducted.[49]

[F: F1, F2, F3] The block F tests, for a given equation [50]
The null is that the block of lags associated with each variable (both LOGJPYUSD$_{-1}$ and LOGJPYUSD$_{-2}$, for example) is excluded/omitted from *a* given *equation* (an equation for dependent variable LOGJPYUSD, for example). See F1 through F3 in Table 13.

Note, however, that "(T)he block F tests ... are not, individually, especially important. (A variable) z can, after all, still affect (another variable) x through the other equations in the (entire) system." (Doan 2007b, p.347) The following two chi-squared tests become thus more relevant and appropriate.

[C1] Chi-squared tests, for the entire model The null is that "*each variable/lag* combination (LOGJPYUSD$_{-1}$, for example) is excluded/omitted from the *entire* model." See C1 in Table 14, which shows that, at any conventional level of significance, every regressor except a constant is to be included in the three-exchange rate VAR model and that the constant is the only one whose "exclusion" null is found not to be rejected at the 1% level of significance.

[C2] Global chi-squared tests, for the entire model The null is that "*all regressors* (that is, *all of the six regressors*, LOGJPYUSD$_{-1}$ through LOGCNYUSD$_{-2}$) across all equations are excluded/omitted with the constant remained." See C2 Table 14: The null is easily rejected.

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[49]An e-mail response of 3/3/2017 from Estima is sincerely appreciated.
[50]Useful references include Doan (2007a, pages 158, 160), Doan (2007b, pages 345, 347) and Estima (2012, p.18).

Implications Combining [C1] and [C2] will lead to the inference that all of lags one and two of the three exhange rates (LOGJPYUSD, LOGEURUSD and LOGCNYUSD) are statistically and managerially important enough to explain the joint behavior of the three daily exchange rates during the sample period V (Monday, June 21, 2010 to Monday, August 10, 2015): The exchange rates are statistically interrelated/interdependent via the estimated VAR(2) model, although **no cointegration relationships among the three are earlier detected** (in Subsection 5.2.2 and subsequently in Subsection 5.3.4). Notice that even LOGCNYUSD which has been controlled carefully by the Chinese central bank and government enters into the picture as a dynamic constituent of the entire, multivariate daily exchange-rate model.

One managerial implication is that, when managerial forecasting of the three daily exchange rates is needed, they are to be considered behaving, especially over a two-day period, jointly in a (multivariate) VAR(2) manner, rather than individually or separately in a univariate time series framework (in Subsections 3 and 4, for example).[51] That is, singling and separating out the Chinese Yuan's exchange rate, in particular, just because of its inflexible nature does not appear appropriate for the managerial forecasting purposes.

A statistical note on two testing methods employed for the ch-squared tests in C1 and C2 in Table 14 The estimated VAR model for Eq. (32) may be written as:

$$\boldsymbol{y}_t = \sum_{l=1}^{2} \hat{\boldsymbol{\Phi}}_l \boldsymbol{y}_{t-l} + \hat{\boldsymbol{\mu}} + \boldsymbol{e}_t \tag{33}$$

where: $\hat{\boldsymbol{\Phi}}_l, \hat{\boldsymbol{\mu}}$ and $\boldsymbol{e}_t$ denote, respectively, estimates of $\boldsymbol{\Phi}_l, \boldsymbol{\mu}$ and $\boldsymbol{u}_t$; in particular, $\boldsymbol{e}_t$ is the residuals vector.

Tests for multiple equations are conducted using a $n_{eq} \times T'^r$ residuals matrix with n_{eq} denoting the number of equations (equal to 3 in the

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[51] For VAR's for forecasting see Doan (2007b, pp.377-381) and for a univariate forecasting see Kojima (2005, especially Section 6) and Doan (2007b, pp.313-339).

present study)[52] (Morrison 1976, p.98):

$$E_{n_{eq} \times T'^r} = (e_1, \cdots, e_t, \cdots, e_{T'^r}) = \begin{bmatrix} e^{1\prime} \\ \vdots \\ e^{i\prime} \\ \vdots \\ e^{n_{eq}\prime} \end{bmatrix} = \begin{bmatrix} e_{11} & \cdots & e_{1,T'^r} \\ & \vdots & \\ e_{n_{eq}1} & \cdots & e_{n_{eq},T'^r} \end{bmatrix}$$

(34)

where: n_{eq}-dimensional residuals vector $e_t = (e_{1t}, \cdots, e_{n_{eq}t})'$; and T'^r-dimensional residuals vector $e^{i\prime} = (e_{i1}, \cdots, e_{i,T'^r})$. Note that a single equation case is obtained by setting $n_{eq} = 1$ in which case one particular T'^r-dimensional residuals vector e^i will be relevant: See Eq. (42).

With e_t being relevant for multiple equations (that is, $n_{eq} > 1$) in the residuals matrix Eq. (34), two testing methods (Doan 2007b, p.350) employed for the ch-squared tests in C1 and C2 in Table 14 may be summarized as follows:

Denoting the maximum (log) likelihood estimates of the residual mean vector and the residual variance/covariance matrix for model $m(= res, unr,$ denoting the restricted model, the unrestricted model), respectively, by $\bar{e}_m$ and Σ_m, the normal likelihood denoted by $L_m(\bar{e}_m, \Sigma_m)$ and the log likelihood maximized denoted by $\log(L_m(\bar{e}_m, \Sigma_m))$ (Morrison 1976, pp.14-16) are written, respectively, as:

$$L(\bar{e}_m, \Sigma_m) = -\frac{1}{(2\pi)^{\frac{n_{eq}T'^r}{2}} |\Sigma_m|^{\frac{T'^r}{2}}} \exp\left\{ -\frac{1}{2}\Sigma_{t=1}^{T'^r}(e_{mt} - \bar{e}_m)'\Sigma^{-1}(e_{mt} - \bar{e}_m) \right\}$$

(35)

$$\log(L(\bar{e}_m, \Sigma_m)) = -\frac{1}{2}\{n_{eq}T'^r \log(2\pi) + T'^r \log|\Sigma_m| + \Sigma_{t=1}^{T'^r}(e_{mt} - \bar{e}_m)'\Sigma_m^{-1}(e_{mt} - \bar{e}_m)\}$$

(36)

(Morrison 1976, p.99) where an n_{eq}-dimensional residuals (time) mean vector

$$\bar{e}_m = (\bar{e}_{m1}, \cdots, \bar{e}_{m,n_{eq}})' \text{ where } \bar{e}_{mi} = \frac{1}{T'^r}\Sigma_{t=1}^{T'^r}e_{mit}, i = 1, ..., n_{eq} \quad (37)$$

---

[52]The number of variables or regressors is not considered in the present statistical note, for focused on is the residuals (from the multiple equations), not the variables or regressors. (n_{eq} is denoted by %nvar in the RATS programs.)

and an n_{eq}-dimensional symmetric, residual variance/covariance matrix

$$\boldsymbol{\Sigma}_m = \frac{1}{T'^r}\Sigma_{t=1}^{T''^r}(\boldsymbol{e}_{mt} - \bar{\boldsymbol{e}}_m)(\boldsymbol{e}_{mt} - \bar{\boldsymbol{e}}_m)' \tag{38}$$

which may be computed, assuming the residual mean vector is a null vector (and thus ignoring $\bar{\boldsymbol{e}}_m$), as follows:

$$\boldsymbol{\Sigma}_m = \frac{1}{T'^r}\Sigma_{t=1}^{T''^r}\boldsymbol{e}_{mt}\boldsymbol{e}'_{mt} = \frac{1}{T'^r}\Sigma_{t=1}^{T''^r}\begin{bmatrix} e_{m1t} \\ \vdots \\ e_{m,n_{eq}t} \end{bmatrix}(e_{m1t},\cdots,e_{m,n_{eq}t})$$

$$= \left\|\frac{1}{T'^r}\Sigma_{t=1}^{T''^r}e_{mit}e_{mjt}\right\|_{n_{eq}\times n_{eq}} = \|s_{mij}\|_{n_{eq}\times n_{eq}} \tag{39}$$

where s_{mij} denotes a residual varaiance/covariance of e_{mit} and e_{mjt} for model m.

   (i) One method is a likelihood ratio test using a likelihood ratio statistic (a chi-squared statistic)

$$(T''^r - c)(|\boldsymbol{\Sigma}_{res}| - |\boldsymbol{\Sigma}_{unr}|) \tag{40}$$

where: $T''^r = 1284$, the number of residuals ($=T'$-the number of lags or the order of VAR$= T - 2$, where the second equality holds since $d+sD = 0$ for the VAR model);[53] $c = 7$, the number of lagged regressors and a constant in each unrestricted equation in a VAR "to improve the small sample properties of the test" (Doan 2007a, p.373; 2007b, p.347); and $|\boldsymbol{\Sigma}_m|$ is the determinant of the <u>residual</u> variance/covariance matrix $\boldsymbol{\Sigma}_m$ as given by (38) above. See the footnotes on the degree of freedom for the chi-squared statistic in Table 14.

   (ii) Exactly the same test result obtains by the second testing method using normal (natural-)log likelihood statistics (Doan 2007a, p.276):[54]

$$-\frac{2(T''^r - c)}{T'^r}\{\log(L(\bar{\boldsymbol{e}}_{res}, \boldsymbol{\Sigma}_{res})) - \log(L(\bar{\boldsymbol{e}}_{unr}, \boldsymbol{\Sigma}_{unr}))\} \tag{41}$$

where $\frac{(T''^r-c)}{T'^r}$ is an adjustment term "to implement the multiplier correction."

---

[53] See the footnote on T, T', and T''^r, respectively, for the raw data, the differenced data and the <u>residuals</u> series for a univariate SARIMA$(p, d, q; P, D, s, Q)$ model in Subsection 3.

[54] That is, the two methods both lead to Table 14 [Source: VAR_VECM_fxdataoutput].

Assuming the residual mean vector is a null vector, the actual computations of (39), (40), (36) and (41) ignore $\bar{e}_m$ (in VAR_VECM_fxdata.prg; Doan 2007a, pages 160, 373, 497), showing that, for each of null hypotheses in Table 14, both statistics, (40) and (41), are exactly equal (as desired).[55]

One remark is in order on C2 in Table 14: T''^r to be used in Eqs. (35) through (41) for the chi-squared tests in C2 is a *smaller* one of the two (that is, the number of rediduals for the unrestricted model and that for the restricted model). With $d + sD = 0$ for the VAR model, the latter model including no lagged regressors has its $T''^r = T' = T$, while the former model including all lagged regressors has a *smaller* $T''^r = T' -$the number of lags or the order of the VAR model $= T' - 2 = T - 2$; also, the residuals series has its $t = 1, \cdots, T''^r$ that corresponds to $1\underline{+}2, \cdots, T$ associated with the raw (undifferenced) data series (as noted on T''^r immediately below Eq. (40) above).[56]

A special case of $n_{eq} = 1$ (that is, a single-equation)[57] in Eqs. (34), (35) and (36) involves no matrix but rather T''^r-dimensional residuals vector $\boldsymbol{e}_m^{1\prime} = (e_{m1t}, \cdots, e_{m1T''^r})$ or simply $\boldsymbol{e}_m' = (e_{m1}, \cdots, e_{mT''^r})$, its (scalar-valued) time mean $\bar{e}_m = \frac{1}{T''^r}\Sigma_{t=1}^{T''^r}e_{mt}$, its (time-invariant) variance $s_{\boldsymbol{e}_m}^2$ and $1/s_{\boldsymbol{e}_m}^2$, which replace, respectively, $\boldsymbol{e}_{mt}', \bar{e}_m, |\boldsymbol{\Sigma}_m|$ and $\boldsymbol{\Sigma}_m^{-1}$ in the two equations (Morrison 1976, pp.15-16).[58] Eq. (36) will then be rewritten, ignoring $\bar{e}_m$ in the second equality immediately below, as:

$$\log(L(\bar{e}_m, s_{\boldsymbol{e}_m}^2)) = -\frac{T''^r}{2}\left\{\log(2\pi) + \log s_{\boldsymbol{e}_m}^2 + \frac{1}{2s_{\boldsymbol{e}_m}^2}\Sigma_{t=1}^{T''^r}(e_{mt} - \bar{e}_m)^2\right\}$$

$$= -\frac{T''^r}{2}\left\{\log(2\pi) + \log\left(\frac{\boldsymbol{e}_m'\boldsymbol{e}_m}{T''^r}\right) + 1\right\}, \tag{42}$$

where the second equality (ignoring $\bar{e}_m$)) may be derived simply by substituting

$$s_{\boldsymbol{e}_m}^2 = \frac{1}{T''^r}\Sigma_{t=1}^{T''^r}e_{mt}^2 = \frac{\boldsymbol{e}_m'\boldsymbol{e}_m}{T''^r} \tag{43}$$

(ignoring $\bar{e}_m$) in the first equality (Doan 2007b, p.178). Using the residual sum of squares (abbreviated as RSS) $\boldsymbol{e}_m'\boldsymbol{e}_m$ in Eq. (43), $m =$

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[55] Details of the test results based on the normal log likelihood statistics (41) and the log likelihood (36) are available from the author upon request.

[56] The RATS program VAR_VECM_fxdata.prg witten to construct Tables 13 and 14 incorporate this technicality.

[57] Still, the number of variables or regressors in the equation is two or more.

[58] Notice the difference in elements and dimension between the two residuals vectors, $\boldsymbol{e}$ and $\boldsymbol{e}_t$.

res, unr, F tests in Table 13 are based on (usual) F statistic:

$$\frac{(e'_{res} e_{res} - e'_{unr} e_{unr})/\mathrm{df}_{num}}{e'_{unr} e_{unr}/\mathrm{df}_{den}} \tag{44}$$

where the degrees of freedom are:[59] df_{num}[=the degree of freedom for the chi-squared variate $e'_{res} e_{res} - e'_{unr} e_{unr}$] $= (T''^r-$the number of parameters (including the constant) in the restricted model) $- (T''^r-$the number of parameters (including the constant) in the unrestricted model)$=$ the number of 'exclusion' restrictions; and $\mathrm{df}_{den} = \mathrm{df}_{unr}$ [the degree of freedom in the unrestricted model, that is, for the chi-squared variate $e'_{unr} e_{unr}$] $= T''^r$ - the number of parameters (including the constant) (see the footnotes on "F1" in Table 13 for details).

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[59]See Johnston (1972, p.207), Maddala (1977, pp.197-198) and Enders (2004, p.183).

Table 13 Tests on Three Differing Nulls of Exclusion/omission: Panel 1 (F tests, F1 through F3);[a] $T = 1286$

The Unrestricted VAR Model (32): Eq. (29) Without the Term $\boldsymbol{\Psi D}_t$

F1:[b]

Dependent Variable LOGJPYUSD

Variable	F-Statistic	Signif
LOGJPYUSD	187911.2304[c]	0.000
LOGEURUSD	2.1782	0.1136699
LOGCNYUSD	5.1729	0.0057872

F2:

Dependent Variable LOGEURUSD

Variable	F-Statistic	Signif
LOGJPYUSD	0.4984	0.6075954
LOGEURUSD	55596.5963	0.0000000
LOGCNYUSD	0.5442	0.5804231

F3:

Dependent Variable LOGCNYUSD

Variable	F-Statistic	Signif
LOGJPYUSD	1.9467	0.1431613
LOGEURUSD	20.9168	0.0000000
LOGCNYUSD	284808.2990	0.0000000

[a]Source: VAR_VECM_fxdataoutput.

[b]Some remarks are in order on technical features of RATS programming (ESTIMATE instruction and LINREG instruction, in particular): The block F test results for "Dependent Variable LOGJPYUSD" (as generated below by ESTIMATE instruction which does NOT display degrees of freedom) can be generated, too, by LINREG instruction (which computes ordinary F statistic, Eq. (44), and does display degrees of freedom), as follows:

F test on the null of the block of two lags (both logJPYUSD{1} and logJPYUSD{2}) being excluded from the LOGJPYUSD equation: F(2,1277)= 187911.23042 with Significance Level 0.00000000; this is exactly the same as that generated by ESTIMATE instruction.

The same hols with the remaining F tests. F test on the null of the block of two lags (both logEURUSD{1} and logEURUSD{2}) being excluded from the LOGJPYUSD equation: F(2,1277)= 2.17816 with Significance Level 0.11366992. F test on the null of the block of two lags (both logCNYUSD{1} and logCNYUSD{2}) being excluded from the LOGJPYUSD equation: F(2,1277)= 5.17294 with Significance Level 0.00578723.

[c]The degree of freedom for the numerator of Eq. (44): $\mathrm{df}_{num} = 2$ [the block of LOGJPYUSD{1} and LOGJPYUSD{2} being excluded]. The degree of freedom for the denominatorof Eq. (44): $\mathrm{df}_{den} (= \mathrm{df}_{unr}) = 1284(T'^r = T' - 2 = T - 2 = 1286 - 2) - 7(6$ **lagged regressors**+the constant) =1277. See the footnote on T, T', and T'^r, respectively, for the raw data, the differenced data and the <u>residuals</u> series for a univariate SARIMA$(p, d, q; P, D, s, Q)$ model in Subsection 3.

Table 14 Tests on Three Differing Nulls of Exclusion/omission: Panel 2 (Chi-squared tests, C1 and C2);[a] $T = 1286$

C1: Test of H0: A System with One Regressor Excluded [A Restricted Model] against H1: A System with All Regressors Included [A Unrestricted Model]:		
A Regressor Excluded	Chi-squared	Stat Signif
LOGJPYUSD{1}	909.7895[b]	0.0000[c]
LOGJPYUSD{2}	11.6677	0.0086
LOGEURUSD{1}	904.9623	0.0000
LOGEURUSD{2}	46.3504	0.0000
LOGCNYUSD{1}	802.6351	0.0000
LOGCNYUSD{2}	11.4797	0.0094
Constant	8.6068	0.0350[d]

C2:[e] Test of H0: A System with No Lagged Regressors (Only with a Constant) against H1: A System with All Regressors Included:		
Regressors Excluded	Chi-squared	Stat Signif
All Lagged Regressors	21999.3180[f]	0.0000[g]

[a]Source: VAR_VECM_fxdataoutput.)

[b]The degree of freedom for the chi-squared statistic is: The total number of regressors, including a constant if included, in the entire *unrestricted* model (Doan 2007b, p.350) - the total number of regressors, including a constant if included, in the entire *restricted* model (Doan 2007a, p.160; 2007b, p.350) =7 regressors×3 equations - 6 regressors×3 equations=3 regressors (=the number of lagged regressors/constant, LOGJPYUSD{1}s, being excluded from the entire model).

[c]The null (of a system without LOGJPYUSD{1}, or of the two log determinants in (40) being equal) is rejected at any conventional level of significance.

[d]The null of a constant being excluded from the entire model is not rejected at 1%.

[e]See the remark made on T'^r below Eq. (40).

[f]The degree of freedom for the chi-squared statistic is with regard to definition the same as that for C1: To be exact, 7 regressors× 3 equations - 1 regressor×3 equations=18 regressors (=6 regressors×3 equations=the number of lagged regressors being excluded from the entire model).

[g]The null (of a system without any lagged regressors, or of the two log determinants in (40) being equal) is rejected at any conventional level of significance.

5.3.4 Roots (Eigenvalues) of the companion matrix

Using the terminology in Harris (1995, p.89),[60] Table 15 and Fig. 23 in Appendix A show that there are $(n \times k = 3 \times 2 =)6$ roots of the

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[60]Harris (1995, p.89):

"The companion matrix seems to be the idea proposed by Juselius(1994)*(Published in 1995).*

These roots are considered since they provide additonal infirmation of how may $(n - r)$ roots are on the unit circle and thus the number of r cointegration relations.

companion matrix; $(n - r = 3 - r =)3$ **roots, which are underlined in the table, are on the unit circle and thus there are found $(r =)0$ cointegration relations: This is exactly the same inference as that derived by the Johansen (1988, 1991) likelihood ratio test in Subsection 5.2.2.**

Table 15 Roots (Eigenvalues) of the Companion Matrix: $T = 1286^a$

Roots (Eigenvalues) of the Companion Matrix:[b]		
Real	Complex	Modulus
1.00	0.00	<u>1.00</u>
1.00	-0.00	<u>1.00</u>
1.00	0.00	<u>1.00</u>
-0.11	0.00	0.11
0.01	-0.03	0.04
0.01	0.03	0.04

[a]Source: VAR_VECM_fxdataoutput.
[b]See Harris (1995, p.89).

6 Concluding Remarks

This paper studies the individual and joint behavior of three daily exchange rates (the Japanese Yen, the Euro and the Chinese Yuan), all against a U.S. dollar, during the period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).

First, logged and then first-differenced, the Japaenese Yen is found to

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The companion matrix is defined by ...

There are $n \times k$ roots of the companion matrix (where: n =the number of potentially endogenous variables, k =the number of lags in AR; in the present example, $n = 5, k = 2$). ...

The moduli of the 3 largest roots are 0.979, 0.918 and 0.918, ... indicating all roots are inside the unit circle, ... This suggests that $n - r = 5 - r = 3$, and thus there are $(r =)$two cointegration relations. ...

The fact that all roots are inside the unit circle is consistent with the endogenous variables comprising I(1) processes, ...

If any of the roots are on or outside the unit circle, this would tend to indicated an I(2) model, requiring second-order differencing to achieve stationarity. For an I(2) model see Box 5.3, pp.93-94."

behave according to either AR(19) or MA(19), while the Euro a white noise.

Second, the Yuan requires an intervention analysis/model incorporating two permanent level shifts invoked by the Chinese central bank's decision of huge devaluation: Two days in a row in mid-August 2015 the largest devaluation was observed in China's system/Yuan in over 20 years . Detected were over 1.8% devaluation from the previous day, on August 11 and nearly 1% devaluation from the previous day, on August 12 (that is, nearly 2% devaluation through the two day period). These estimates both approximate the corresponding daily (actual, exact) rates of change in the Yuan exchange rate.

Third, noting that the period V (Monday, June 21, 2010 - Monday, August 10, 2015), turns out the longest period of time when the Yuan was continuously less managed/controlled by the central bank in China under (managed) flexible exchange rate system (see Section 1 referring to Tables 1 and 2), the VAR modeling detects for the period no cointegration relationships among the three daily exchange rates, and yet the chi-squared tests for their unrestricted VAR models (that is, VAR models with no a priori restrictions/cointegrations) show that even the China's Yuan exchange rate which has been controlled carefully by the Chinese central bank and government enters into the picture as a statistically significant constituent of the multivariate daily exchange-rate model.

Thus, singling and separating out the Yuan's exchange rate, in particular, just because of its inflexible nature does not appear appropriate, although no cointegration relationships exist either a priori or a posteriori among the three daily exchange rates (the Japanese Yen, the Euro and the Chinese Yuan). The VAR modeling of the three may be still meaningful for the managerial forecasting purposes.

Similar time series econometric study remains for two previous periods I and III, during which China employed (managed) flexible exchange rate system; during period I in particular a multiple AOs will be most likely detected (see Table 1 and Figs. 3 and 4). Contrasting the results among periods I, III (in the future work) and V (in the present paper) will be one topic to be studied.
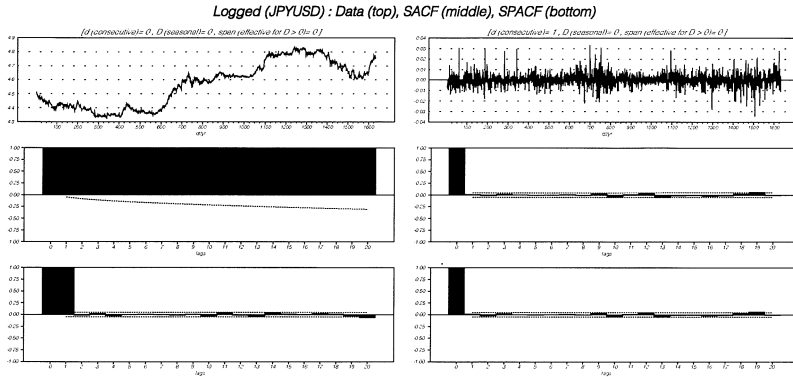
Appendices

A   Figure Appendix

Figure 9   Identification for Logged Daily JPY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).
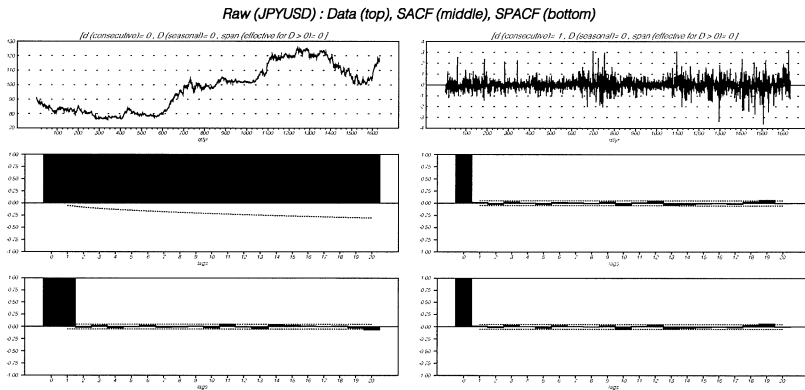

Figure 10   Identification for Raw Daily JPY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).
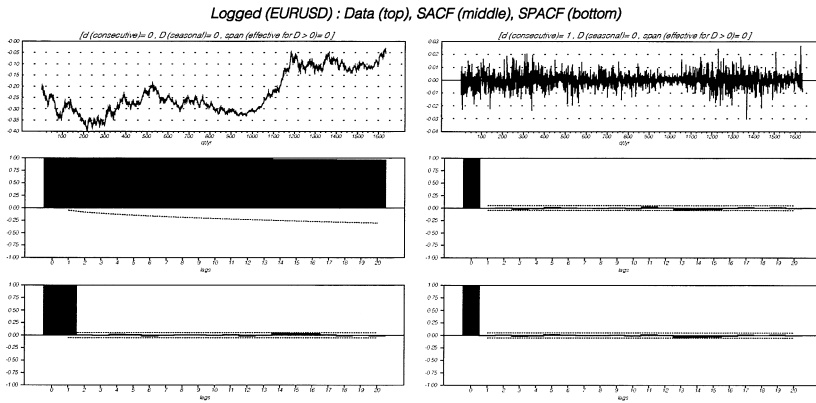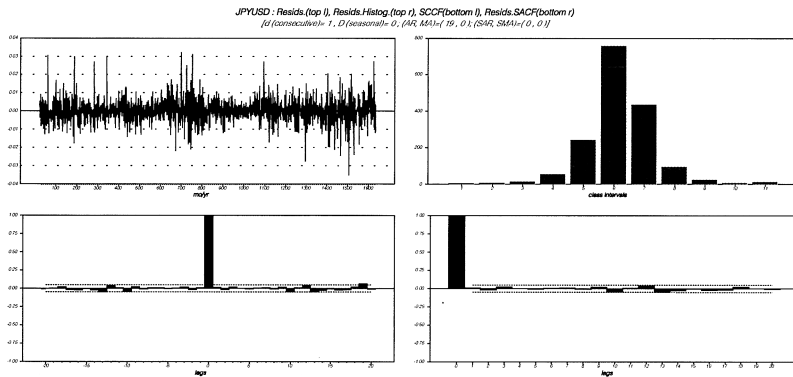
Logged (EURUSD) : Data (top), SACF (middle), SPACF (bottom)

Figure 11 Identification for Logged Daily EUR/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).

JPYUSD : Resids.(top l), Resids.Histog.(top r), SCCF(bottom l), Resids.SACF(bottom r)
[d (consecutive)= 1, D (seasonal)= 0, (AR, MA)=(19 , 0); (SAR, SMA)=(0 , 0)]

Figure 12 AR Model without a Constant: Estimation for First-differenced, Logged Daily JPY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).
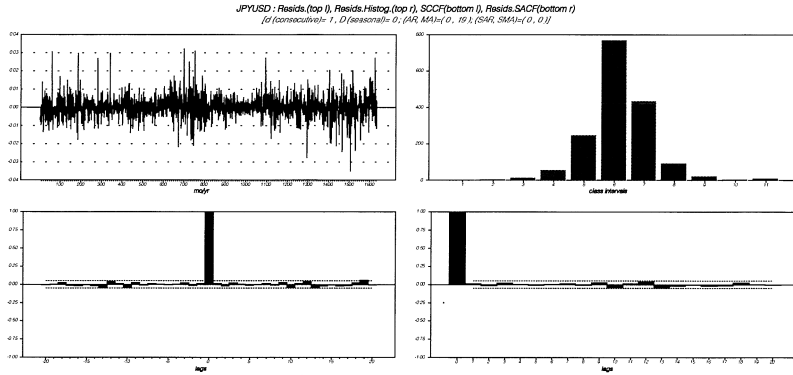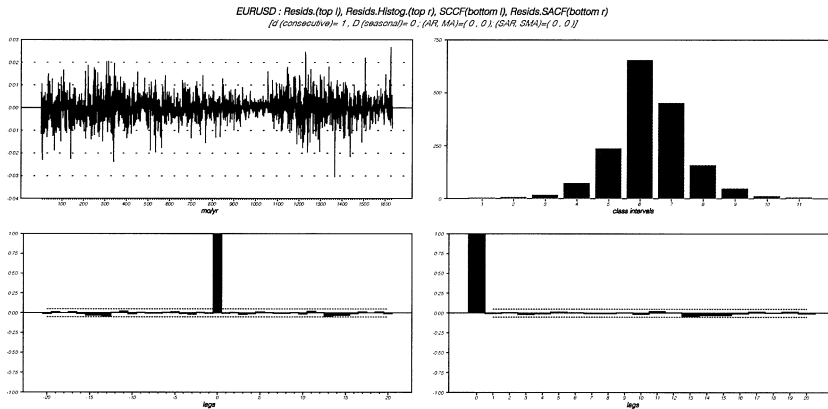
Figure 13   MA Model without a Constant: Estimation for First-differenced, Logged Daily JPY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).

Figure 14   White Noise Model without a Constant: Estimation for First-differenced, Logged Daily EUR/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).
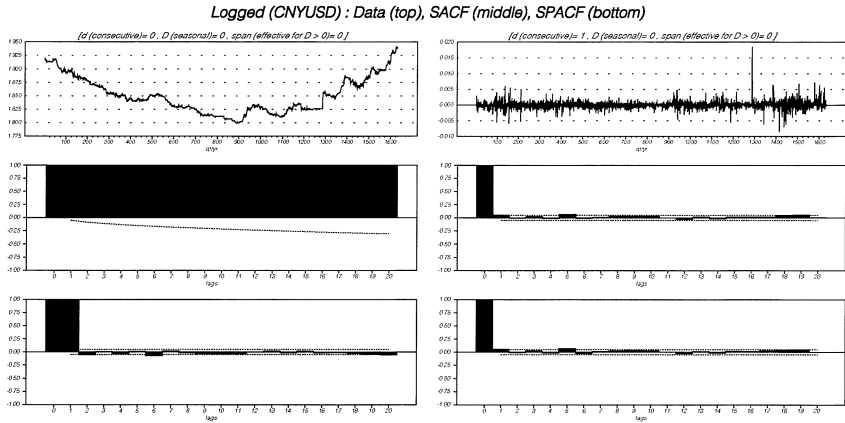
Logged (CNYUSD) : Data (top), SACF (middle), SPACF (bottom)

Figure 15 Identification for Logged Daily CNY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).

CNYUSD : Resids.(top l), Resids.Histog.(top r), SCCF(bottom l), Resids.SACF(bottom r)
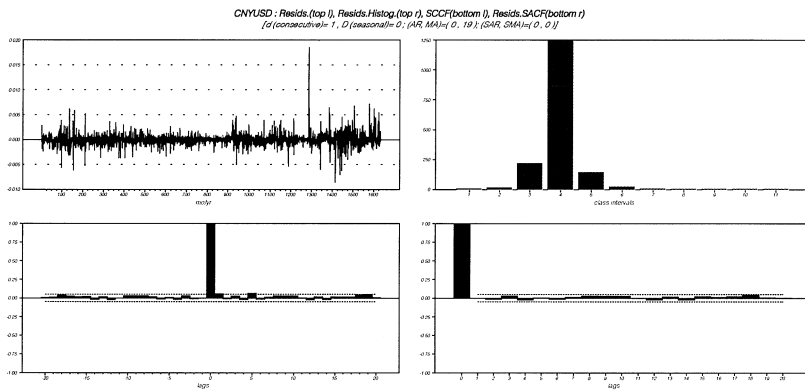[d (consecutive)= 1 , D (seasonal)= 0 ; (AR, MA)=(0 , 19); (SAR, SMA)=(0 , 0)]

Figure 16 Estimation for First-differenced, Logged Daily CNY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).
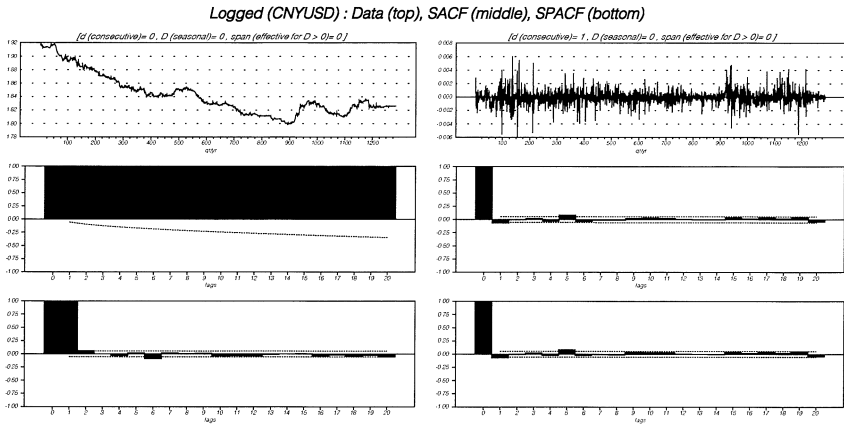
Logged (CNYUSD) : Data (top), SACF (middle), SPACF (bottom)

Figure 17 Identification for Logged Daily CNY/USD, Period V (Monday, June 21, 2010 - Monday, August 10, 2015).

CNYUSD (Intervention Model) : Resids.(top l), Resids.Histog.(top r), SCCF(bottom l), Resids.SACF(bottom r)
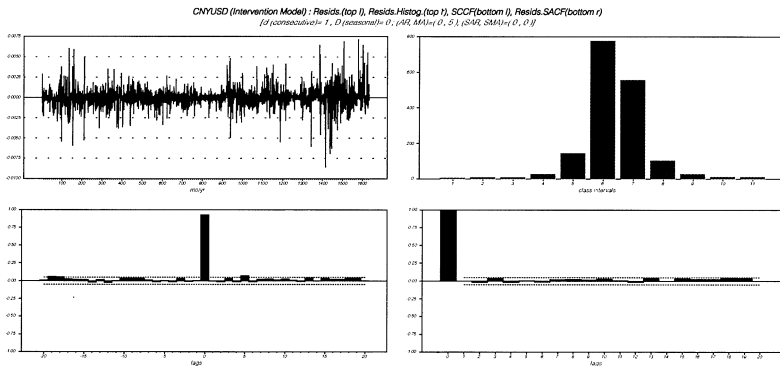
Figure 18 [First Version] Estimated Intervention Model (without a Constant) with θ_1 and θ_5 and permanent level shifts on August 11 and 12, 2015: First-differenced, Logged Daily CNY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).

CNYUSD (Intervention Model) : Resids.(top l), Resids.Histog.(top r), SCCF(bottom l), Resids.SACF(bottom r)
[d (consecutive)= 1, D (seasonal)= 0 ; (AR, MA)=(0, 5); (SAR, SMA)=(0, 0)]

Figure 19 [Second Version] Estimated Intervention Model (without a Constant) with θ_5 and permanent level shifts on August 11 and 12, 2015: First-differenced, Logged Daily CNY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).
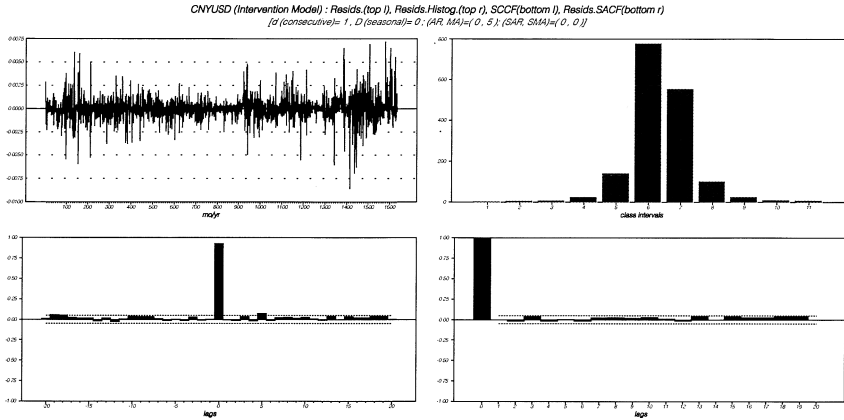
CNYUSD (Intervention Model) : Resids.(top l), Resids.Histog.(top r), SCCF(bottom l), Resids.SACF(bottom r)
[d (consecutive)= 1, D (seasonal)= 0 ; (AR, MA)=(19, 5); (SAR, SMA)=(0, 0)]

Figure 20 [Third, Final Version] Estimated Intervention Model (without a Constant) with ϕ_{19} and θ_5 and permanent level shifts on August 11 and 12, 2015: First-differenced, Logged Daily CNY/USD, Period V through 2016 (Monday, June 21, 2010 - Friday, December 30, 2016).
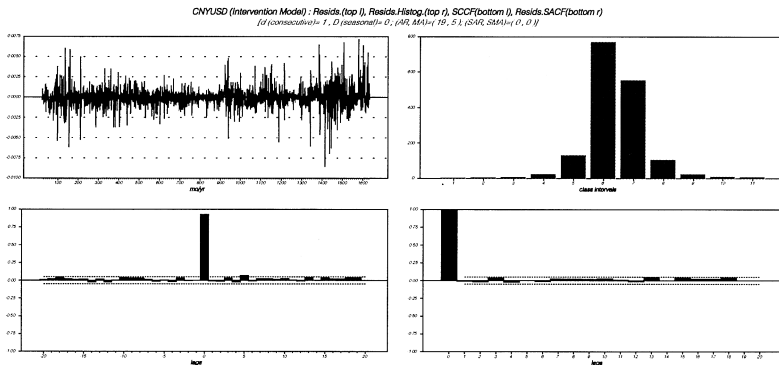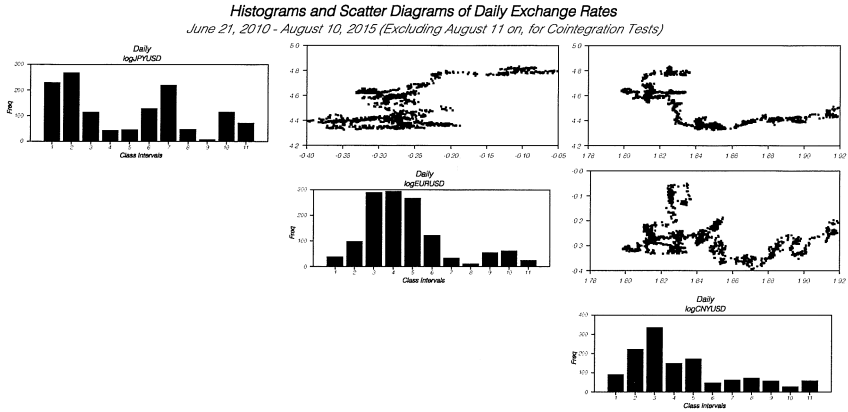
Histograms and Scatter Diagrams of Daily Exchange Rates
June 21, 2010 - August 10, 2015 (Excluding August 11 on, for Cointegration Tests)

Figure 21 Histograms and Scatter Diagrams of Logged Daily Exchange Rates, Period V (Monday, June 21, 2010 - Monday, August 10, 2015).

Histograms and Scatter Diagrams of Daily Exchange Rates
June 21, 2010 - August 10, 2015 (Excluding August 11 on, for Cointegration Tests)
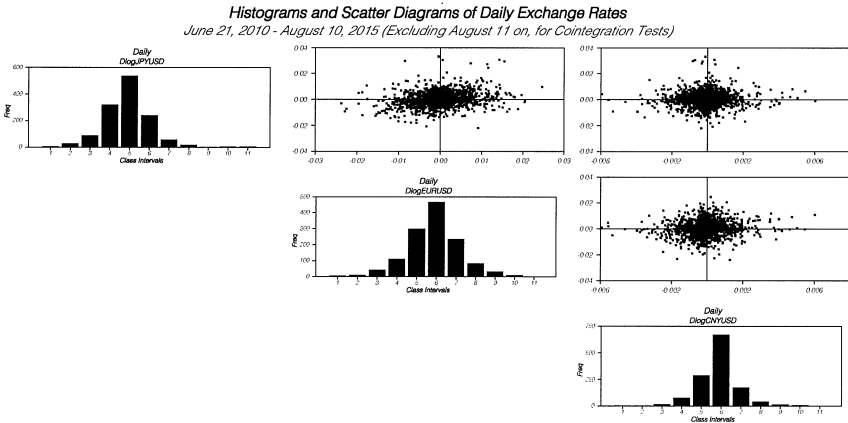
Figure 22 Histograms and Scatter Diagrams of First Differences of Logged Daily Exchange Rates, Tuesday, June 22, 2010 - Monday, August 10, 2015. Note: For the plot of first-differenced, logged exchange rates for Tuesday, June 22, 2010 - Friday, December 30, 2016, see Fig. 7; see Subsection 3.1 for the economic interpretation of first-differenced, logged series as a rate of change.
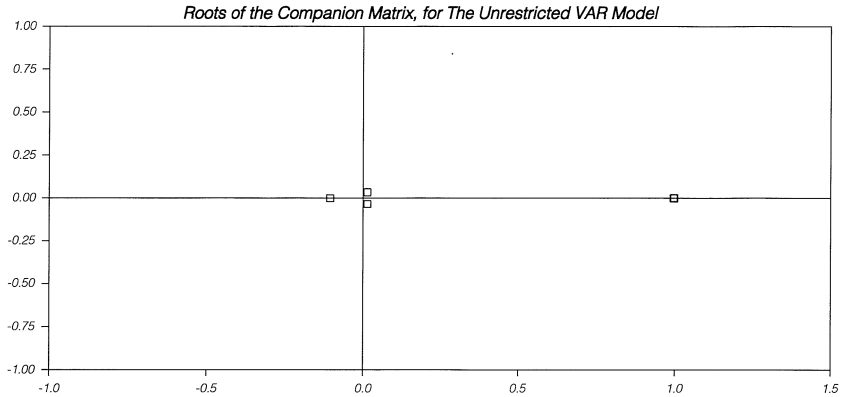
Roots of the Companion Matrix, for The Unrestricted VAR Model

Figure 23   Roots (Eigenvalues) of the Companion Matrix for Logged Daily Exchange Rates, Period V (Monday, June 21, 2010 - Monday, August 10, 2015).

B   Table Appendix

Table 16 lists the source of each table, which is available from the author upon request:

Table 16   Sources of Tables

Table Number	Source
1 and 2	BJidentify_fixdata_Jan31994-Dec311996output, BJidentify_fixdata_Jul212005-Jul312008output and BJidentify_fxdataoutput
4	BJetimate_jpyModelBoutput
5	BJetimate_jpyModelB-MAoutput
6	BJetimate_eurModelBoutput
7	BJetimate_cnyModelB-MAoutput
8	BJetimate_cnyModelB-MAoutput_forIdentify1-1286
9	BJetimate_cnyModelB-MA_IntrvModel2output
10	BJetimate_cnyModelB-ARMA_IntrvModeloutput
13, 14 and 15	VAR_VECM_fxdataoutput
Table Appendix:	
17 through 24	CointegrTests_output
25 and 26	CATS_fxdataOutput2_nlags=2
27	CATS_fxdataOutput2
28	CATS_fxdataOutput3
29	CATS_fxdataOutput
30 and 31	VARLAG_3exchr_output

Table 17 Augmented DF Test: $T = 1286^a$

Dickey-Fuller Unit Root Test, Series LOGJPYUSD
Regression Run From 23^b to 1286
Observations 1265^c
With intercept and trend^d
Using 21 lags on the differences^e

Sig Level Crit Value
1%(**) -3.97043
5%(*) -3.41580
10% -3.12982
T-Statistic -2.45555

Dickey-Fuller Unit Root Test, Series LOGEURUSD
Same as Above
T-Statistic -1.69295

Dickey-Fuller Unit Root Test, Series LOGCNYUSD
Same as Above
T-Statistic -1.71981

^{a}See MacDonald and Marsh (1994,p.33), Harris (1995, pp.32-39), Enders (2004, pp.181-199) and Doan (2007b, pp.249-250). Source: CointegrTests_output.

^{b}This equals $1 + d + (p - 1) = 1 + 1 + 21$ (see Table 3). This is consistent with the following observation. With $p = 22$ in Eq. (28), $\psi^*_{p-1}\Delta y_{t-p+1} = \psi^*_{21}\Delta y_{t-21}$, and thus the equation for $t = 23$ will contain y_1:

$$y_{23} - y_{22} = a_0 + \psi^* y_{22} + \psi^*_1 (y_{22} - y_{21}) + \cdots + \psi^*_{21}(y_2 - y_1) + a_2 23 + u_{23},$$
$$(45)$$

meaning that the <u>residuals</u> (the estimated u_ts) start at $t = 23$. This is evidenced by the <u>residuals</u> computed (by LINREG in @DFUNIT, written in RATS): ENTRY 1 2 ... 22 *23* ... 1286; RESIDSHIRAO NA NA ... NA *-0.001774369187* ... 0.001891308529. See also the footnote immediately below.

^{c}The usual effective number of observations (that is, %nobs defined by LINREG) is $T''^r = T' - (p - 1) = T - d - (p - 1) = 1286 - 1 - 21 = 1264$, for the <u>residuals</u> (see Table 3); this is consistent with "Regression Run From 23 to 1286." "Observations" here displayed by @DFUNIT is, however, that usual effective number **plus one**: $T'^r + 1$. This "**plus-one**" figure is used when computing critical values in @DFUNIT whose RATS program [saved in the folder 'Business Forecasting&PanelDtAnlys (Incld. Anderson): Yr2016o_new research: Published in 2018: MacRATS] is available from the author upon request.

^{d}See Eq. (28).

^{e}See Eqs. (27) through (28).

Table 18 Phillips-Perron Test: $T = 1286^a$

| Phillips-Perron Test for a Unit Root for LOGJPYUSD |
| Regression Run From 2^b to 1286 Observations 1285^c |
| With intercept and trend |

| Sig Level Crit Value |
| 1%(**) -3.970324 |
| 5%(*) -3.415752 |
| 10% -3.129785 |
| Lags Statistic |
| 4^d -2.89128 |

| Phillips-Perron Test for a Unit Root for LOGEURUSD |
| Same as Above |
| Lags Statistic |
| 4 -1.98497 |

| Phillips-Perron Test for a Unit Root for LOGCNYUSD |
| Same as Above |
| Lags Statistic |
| 4 -1.41067 |

^{a}See Harris (1995, pp.33-34), Enders (2004, p.229) and Doan (2007b, pp.249-250). Source: CointegrTests_output.

^{b}This is a default set by RATS.

^{c}This is a default set by RATS.

^{d}In RATS: LAGS=number of lags in spectral estimation window [4].

Table 19 Engle-Granger (1987) Test: Stage 1: $T = 1286^a$

| Linear Regression - Estimation by Least Squares |
| Dependent Variable LOGJPYUSD |
| Usable Obs. 1286 DF 1282 |
| Centered R**2 0.853704 R Bar **2 0.853361 |
| Omitted. |
| Variable Coeff Std Error T-Stat Signif |
| 1. LOGEURUSD -0.135212309 0.033506034 -4.03546 0.00005772 |
| 2. LOGCNYUSD 2.864094849 0.120530068 23.76249 0.00000000 |
| 3. Constant -1.147504446 0.233585028 -4.91258 0.00000101 |
| 4. TRND 0.000565742 0.000011304 50.04779 0.00000000 |

^{a}Source: CointegrTests_output.

Table 20 Engle-Granger (1987) Test: Stage 2a: $T = 1286^a$

Linear Regression - Estimation by Least Squares
Dependent Variable DU[b]
Usable Obs. 1263[c] DF 1240
Centered R**2 0.026643 R Bar **2 0.009374
Uncentered R**2 0.026730 T x R**2 33.760
Mean of Dependent Variable -0.000063770
Std Error of Dependent Variable 0.006763426
Standard Error of Estimate 0.006731651
Sum of Squared Residuals 0.0561907616
Log Likelihood 4535.66727
Durbin-Watson Statistic 1.997635
Variable Coeff Std Error T-Stat Signif
1. U{1} -0.007886035 0.003398998 -2.32011 0.02049683
2. DU{1}[d] -0.069128807 0.028359174 -2.43762 0.01492418
3. DU{2} -0.026690766 0.028420450 -0.93914 0.34784205
4. DU{3} 0.027222289 0.028414662 0.95804 0.33823099
5. DU{4} 0.022544444 0.028360214 0.79493 0.42680513
6. DU{5} 0.056725149 0.028322410 2.00284 0.04541222
7. DU{6} 0.001321923 0.028263306 0.04677 0.96270272
8. DU{7} 0.023867381 0.028221791 0.84571 0.39787909
9. DU{8} -0.027613551 0.028223792 -0.97838 0.32807800
10. DU{9} 0.022362734 0.028211658 0.79268 0.42811762
11. DU{10} -0.014173817 0.028223005 -0.50221 0.61561043
12. DU{11} 0.016416771 0.028172496 0.58272 0.56018551
13. DU{12} -0.000244527 0.028178109 -0.00868 0.99307752
14. DU{13} -0.009728139 0.028164186 -0.34541 0.72984614
15. DU{14} -0.038018941 0.028157073 -1.35024 0.17718386
16. DU{15} 0.005303119 0.028151716 0.18838 0.85061242
17. DU{16} -0.060761609 0.028139008 -2.15934 0.03101527
18. DU{17} -0.017064422 0.028174829 -0.60566 0.54485009
19. DU{18} 0.004388230 0.028138035 0.15595 0.87609491
20. DU{19} 0.076879450 0.028133528 2.73266 0.00637172
21. DU{20} 0.030723965 0.028214226 1.08895 0.27638623
22. DU{21} 0.029155369 0.028242432 1.03233 0.30212117
23. DU{22} -0.018130207 0.028121114 -0.64472 0.51922879

[a]Source: CointegrTests_output.

[b]The first-differenced residuals: $DU_t = (1-B)e_t = e_t - e_{t-1}$. (In RATS program EGTEST.src, "set du = u-u{1}" where u denotes a residual e_t and u{1} e_{t-1}.)

[c]This equals $T''^r = T' - \max\{p, sP\} = T - d\underline{-p} = 1286 - 1\underline{-22}$. See Table 3.

[d]$DU_{t-1} = (1 - B)e_{t-1} = e_{t-1} - e_{t-2}$.

Table 21 Engle-Granger (1987) Test: Stage 2b: $T = 1286^a$

Engle-Granger Cointegration Test
Test Statistic -2.32011
Critical Values 1% 5% 10%
-4.68 -4.13 -3.84

^{a}Source: CointegrTests_output.

Table 22 Engle-Granger (1987) Test: Stage 1: $T = 1286^a$

Linear Regression - Estimation by Least Squares
Dependent Variable LOGJPYUSD
Usable Obs. 1286 DF 1283
Centered R**2 0.567869 R Bar **2 0.567196
Uncentered R**2 0.999512 T x R**2 1285.372
Mean of Dependent Variable 4.5275853025
Std Error of Dependent Variable 0.1523081965
Standard Error of Estimate 0.1002003161
Sum of Squared Residuals 12.881452604
Regression F(2,1283) 843.0044
Significance Level of F 0.00000000
Log Likelihood 1135.29774
Durbin-Watson Statistic 0.006069
Variable Coeff Std Error T-Stat Signif
1. LOGEURUSD 1.055368621 0.040536793 26.03483 0.00000000
2. LOGCNYUSD -2.468767696 0.096780363 -25.50897 0.00000000
3. Constant 9.354140779 0.176313104 53.05414 0.00000000

^{a}Source: CointegrTests_output.

Table 23 Engle-Granger (1987) Test: Stage 2a: $T = 1286^a$

Linear Regression - Estimation by Least Squares
Dependent Variable DU
Usable Obs. 1263 DF 1240
Centered R**2 0.017494 R Bar **2 0.000063
Uncentered R**2 0.017502 T x R**2 22.105
Mean of Dependent Variable -0.000021868
Std Error of Dependent Variable 0.007782447
Standard Error of Estimate 0.007782203
Sum of Squared Residuals 0.0750977188
Log Likelihood 4352.50839
Durbin-Watson Statistic 1.999513
Variable Coeff Std Error T-Stat Signif
1. U{1} -0.003522276 0.002220492 -1.58626 0.11293544
2. DU{1} -0.008006239 0.028383519 -0.28207 0.77793422
Omitted.
9. DU{8} -0.055785812 0.028323180 -1.96962 0.04910461
10. DU{9} 0.067541591 0.028360920 2.38150 0.01739237
Omitted.
23. DU{22} -0.005317816 0.028264754 -0.18814 0.85079534

^{a}Source: CointegrTests_output.

Table 24 Engle-Granger (1987) Test: Stage 2b: $T = 1286^a$

Engle-Granger Cointegration Test
Test Statistic -1.58626
Critical Values 1% 5% 10%
-4.31 -3.75 -3.46

^{a}Source: CointegrTests_output.

Table 25 Johansen (1988, 1991) Likelihood Ratio Test, with a Drift Term:[a] $T = 1286$[b]

@cats(lags=nlags,dettrend=drift) 1 1286 ;* = June 21, 2010 - Monday, August 10, 2015

\# logJPYUSD logEURUSD logCNYUSD

CATS for RATS version 2 - 02/26/2017 14:10

MODEL SUMMARY

Sample: 1 to 1286 (1286 observations)

Effective Sample: 3[c] to 1286 (1284 observations[d])

Obs. - No. of variables: 1277

System variables: LOGJPYUSD LOGEURUSD LOGCNYUSD

Constant/Trend: Unrestricted Constant

Lags in VAR: 2

I(2) analysis not available for the specified model.

The unrestricted estimates:

Omitted.

I(1)-ANALYSIS: Rank Test Statistics

p-r[e] r Eig.Value Trace Trace* Frac95 P-Value P-Value*

3 0 0.013 22.150 22.075 29.804 0.299 0.304

2 1 0.004 5.311 4.120 15.408 0.775 0.888

1 2 0.001 0.691 0.503 3.841 0.406 0.478

---

[a]For a drift term see Eq. (24) in Subsection 5.2.1.

[b]The number of lags to be considered in the present and the following tables is set equal to 2, as will be shown in Subsection 5.3 to be appropriate for VAR modeling. Source: CATS_fxdataOutput2_nlags=2.

[c]The beginning of the *effective* sample is readily computed based on Eq. (30): With $L = 2$, that is, $p = 2$, in Eq. (30), the equation for $t = 3$ will contain $\boldsymbol{y}_1$:

$$\boldsymbol{y}_3 - \boldsymbol{y}_2 = \boldsymbol{\Phi}_1^{\Delta}(\boldsymbol{y}_2 - \boldsymbol{y}_1) + \boldsymbol{\Pi}\boldsymbol{y}_2 + \boldsymbol{u}_3, \tag{46}$$

meaning that the residuals (the estimated u_ts) start at $t = 3$.

[d]As is obvious from the footnote immediately above, the effective number of observations equals $T-p = 1286-2$, which is earlier denoted by T'^r for the residuals: See Table 3.

[e]The symbol p is not p, the order of VAR, bur rather n, in the text, while r is the same as r in the text.

Table 26 Johansen (1988, 1991) Likelihood Ratio Test, with No Deterministic Term[a]

@cats(lags=nlags,dettrend=none) 1 1286
CATS for RATS version 2 - 02/26/2017 14:13
MODEL SUMMARY
Sample: 1 to 1286 (1286 observations)
Effective Sample: 3 to 1286 (1284 observations)
Obs. - No. of variables: 1278
System variables: LOGJPYUSD LOGEURUSD LOGCNYUSD
Constant/Trend: None
Lags in VAR: 2
The unrestricted estimates:
Omitted.
I(1)-ANALYSIS: Rank Test Statistics
p-r r Eig.Value Trace Trace* Frac95 P-Value P-Value*
3 0 0.012 23.682 23.605 24.214 0.059 0.060
2 1 0.005 7.552 4.750 12.282 0.275 0.599
1 2 0.001 1.032 0.410 4.071 0.361 0.592

[a]Source: CATS_fxdataOutput_nlags=2.

Table 27 Johansen (1988, 1991) Likelihood Ratio Test, with a Drift Term: $T = 1286^a$

```
@cats(lags=nlags,dettrend=drift) 1 1286 ;* = June 21, 2010 -
Monday, August 10, 2015
# logJPYUSD logEURUSD logCNYUSD
```
```
CATS for RATS version 2 - 02/25/2017 13:27
MODEL SUMMARY
Sample: 1 to 1286 (1286 observations)
Effective Sample: 23ᵇ to 1286 (1264 observationsᶜ)
Obs. - No. of variables: 1197
System variables: LOGJPYUSD LOGEURUSD LOGCNYUSD
Constant/Trend: Unrestricted Constant
Lags in VAR: 22
```
```
I(2) analysis not available for the specified model.
The unrestricted estimates:
BETA(transposed)
LOGJPYUSD LOGEURUSD LOGCNYUSD
Beta(1) 1.431 -2.189 40.961
Beta(2) -8.962 16.439 -13.488
Beta(3) 4.817 8.504 17.340
ALPHA
Alpha(1) Alpha(2) Alpha(3)
DLOGJP -0.000 0.000 -0.000
(-3.020) (0.907) (-0.202)
DLOGEU -0.000 -0.000 -0.000
(-1.357) (-1.303) (-0.302)
DLOGCN -0.000 -0.000 0.000
(-2.630) (-0.798) (0.282)
PI
LOGJPYUSD LOGEURUSD LOGCNYUSD
DLOGJP -0.002 0.003 -0.022
(-1.307) (1.063) (-3.000)
DLOGEU 0.001 -0.003 -0.007
(0.806) (-1.127) (-0.931)
DLOGCN 0.000 -0.000 -0.003
(0.462) (-0.267) (-1.981)
Log-Likelihood = 21734.970
```
```
Cannot compute Bartlett Small Sample Correction
due to the number of lags in use.
Uncorrected values will be used instead.
I(1)-ANALYSIS: Rank Test Statistics
p-r r Eig.Value Trace Trace* Frac95 P-Value P-Value*
3 0 0.012 19.495 19.495 29.804 0.469 0.469
2 1 0.003 4.124 4.124 15.408 0.887 0.887
1 2 0.000 0.203 0.203 3.841 0.652 0.652
```

^{a}The number of lags to be considered in the three tables below is set equal to $12 \times (T/100)^{1/4}$, which is 22 with $T = 1286$ (see the beginning of Subsection 5.2). Source: CATS_fxdataOutput2.

^{b}This is readily computed based on Eq. (30): With $L = 22$, that is, $p = 22$, in Eq. (30), the equation for $t = 23$ containing y_1 will be:

$$y_{23} - y_{22} = \Phi_1^\Delta (y_{22} - y_{21}) + \cdots + \Phi_{21}^\Delta (y_2 - y_1) + \Pi y_{22} + u_{23}. \tag{47}$$

^{c}As is obvious from the footnote immediately above, the effective number of observations equals $T - p = 1286 - 22$.

Table 28   Johansen (1988, 1991) Likelihood Ratio Test, with a CIDrift Term: $T = 1286^a$

@cats(lags=nlags,dettrend=cidrift) 1 1286
CATS for RATS version 2 - 02/25/2017 21:20
MODEL SUMMARY
Sample: 1 to 1286 (1286 observations)
Effective Sample: 23 to 1286 (1264 observations)
Obs. - No. of variables: 1196
System variables: LOGJPYUSD LOGEURUSD LOGCNYUSD
Constant/Trend: Restricted Trend
Lags in VAR: 22
The unrestricted estimates:
Omitted.
Cannot compute Bartlett Small Sample Correction
due to the number of lags in use.
Uncorrected values will be used instead.
I(1)-ANALYSIS: Rank Test Statistics
p-r r Eig.Value Trace Trace* Frac95 P-Value P-Value*
3 0 0.012 27.069 27.069 42.770 0.680 0.680
2 1 0.006 11.458 11.458 25.731 0.844 0.844
1 2 0.003 3.918 3.918 12.448 0.752 0.752

   ^{a}Source: CATS_fxdataOutput3.

Table 29   Johansen (1988, 1991) Likelihood Ratio Test, with No Deterministic Term: $T = 1286^a$

@cats(lags=nlags,dettrend=none) 1 1286
CATS for RATS version 2 - 02/25/2017 21:13
MODEL SUMMARY
Sample: 1 to 1286 (1286 observations)
Effective Sample: 23 to 1286 (1264 observations)
Obs. - No. of variables: 1198
System variables: LOGJPYUSD LOGEURUSD LOGCNYUSD
Constant/Trend: None
Lags in VAR: 22
The unrestricted estimates:
Omitted.
Cannot compute Bartlett Small Sample Correction
due to the number of lags in use.
Uncorrected values will be used instead.
I(1)-ANALYSIS: Rank Test Statistics
p-r r Eig.Value Trace Trace* Frac95 P-Value P-Value*
3 0 0.008 18.489 18.489 24.214 0.229 0.229
2 1 0.006 8.634 8.634 12.282 0.193 0.193
1 2 0.001 1.321 1.321 4.071 0.293 0.293

    ^{a}Source: CATS_fxdataoutput.

Table 30 Setting the Lag Length for VAR:[a] Panel 1 ([A] and [B]); $T = 1286$

[A] H1: longernlags= 2 , H0: shorternlags= 1
Using RATIO:
Log Determinants are -34.234474 -34.183470
Chi-Squared(9)[b] = 65.132634 with Significance Level 0.00000000
Using calculated statistic:
Chi-Squared(9)= 72.495898 with Significance Level 0.00000000
Using VARLagSelect procedure:
Lags AICC
0 -10899.677
1 -32936.076
2 -32983.321*
Lags AIC SBC LR Test P-Value
1 -25.6453 -25.5972
2 -25.6879* -25.6038* 47.2292 0.0000

[B] H1: longernlags= 5 , H0: shorternlags= 2
Using RATIO:
Log Determinants are -34.245756 -34.233826
Chi-Squared(27)[c]= 15.090870 with Significance Level 0.96813814
Using calculated statistic:
Chi-Squared(27)= 14.271093 with Significance Level 0.97846471
Using VARLagSelect procedure:
Lags AICC
0 -10893.795
1 -32858.745
2 -32905.412*
3 -32891.695
4 -32878.046
5 -32865.612
Lags AIC SBC LR Test P-Value
1 -25.6453 -25.5972
2 -25.6879* -25.6038* 47.2292 0.0000
3 -25.6765 -25.5563 -22.1091 NA
4 -25.6674 -25.5112 -19.0517 NA
5 -25.6562 -25.4640 -21.5501 NA

(Continued to Next Table)

[a]Doan (2007b, pp.348-349) gives an example of testing a lag length, whose programming is applied in the present and the following tables. Source: VARLAG_3exchr_output.

[b]The degree of freedom 9 = the total number of (lagged) regressors under the alternative H_1—that under the null $H_0 = 2 \times 3 \times 3 - 1 \times 3 \times 3 = 18 - 9 =$ the number of parameters *excluded* in H_0 as against H_1.

[c]The degree of freedom 27 = the total number of (lagged) regressors under the alternative H_1—that under the null $H_0 = 5 \times 3 \times 3 - 2 \times 3 \times 3 = 45 - 18 =$ the number of parameters *excluded* in H_0 as against H_1.

Table 31 Setting the Lag Length for VAR:[a] Panel 2 ([C]); $T = 1286$

[C] H1:longernlags= 20 , H0: shorternlags= 2
Using RATIO:
Log Determinants are -34.375754 -34.251438
Chi-Squared(162)[b]= 149.800918 with Significance Level 0.74480850
Using calculated statistic:
Chi-Squared(162)= 170.242524 with Significance Level 0.31303421
Using VARLagSelect procedure:
Lags AICC
0 -10850.279
1 -32491.464
2 -32542.543*
3 -32528.986
4 -32516.758
5 -32505.232
6 -32500.763
7 -32488.915
8 -32478.483
9 -32471.280
10 -32461.394
11 -32451.120
12 -32437.281
13 -32425.072
14 -32413.611
15 -32402.650
16 -32390.429
17 -32382.282
18 -32373.453
19 -32359.608
20 -32356.814
Lags AIC SBC LR Test P-Value
1 -25.6453 -25.5972
2 -25.6879* -25.6038* 47.2292 0.0000
3 -25.6765 -25.5563 -22.1091 NA
4 -25.6674 -25.5112 -19.0517 NA
5 -25.6562 -25.4640 -21.5501 NA
6 -25.6516 -25.4234 -13.0232 NA
7 -25.6464 -25.3823 -13.7499 NA
8 -25.6413 -25.3412 -13.5027 NA
9 -25.6447 -25.3088 -2.4198 NA
10 -25.6375 -25.2657 -15.9981 NA
11 -25.6296 -25.2220 -16.7199 NA
12 -25.6170 -25.1736 -22.6656 NA
13 -25.6126 -25.1333 -12.1372 NA
14 -25.6015 -25.0865 -20.4071 NA
15 -25.5913 -25.0407 -19.1983 NA
16 -25.5825 -24.9962 -17.3320 NA
17 -25.5744 -24.9524 -16.3066 NA
18 -25.5722 -24.9146 -8.7547 NA
19 -25.5621 -24.8689 -18.6194 NA
20 -25.5583 -24.8295 -10.5691 NA

[a]Source: VARLAG_3exchr_output.

[b]The degree of freedom 162 = the total number of (lagged) regressors under the alternative H_1 — that under the null $H_0 = 20 \times 3 \times 3 - 2 \times 3 \times 3 = 180 - 18 = $ the number of parameters *excluded* in H_0 as against H_1.

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